Male and female life expectancy co-move — even when they diverge*

Preliminary version; please contact the authors for latest version before citing. Comments appreciated.

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Abstract

We develop a framework for mortality sex differentials in time series, that of co-movement/anti-movement and convergence/divergence. We apply this framework to data from the Human Mortality Database (HMD). We use the nonparametric test of Goodman and Grunfeld (1961) to test co-movement between male and female life expectancy. For every country in the HMD (except two with very short spans of data, Chile and Luxembourg), male and female mortality statistically co-move. This applies even in cases, including ones such as Russia that are well-discussed in the literature, which show divergence between the sexes. That is to say, even when the life expectancy sex differential increases, male and female life expectancy co-move. The results are robust to subsetting with the same time-window width for all countries. The sex divergence that has been so well-discussed needs to be (re-)considered in light of the fact that male and female life expectancy virtually always co-move, reflecting overall societal mortality factors.

*Approx. X,XXX words, all-inclusive.

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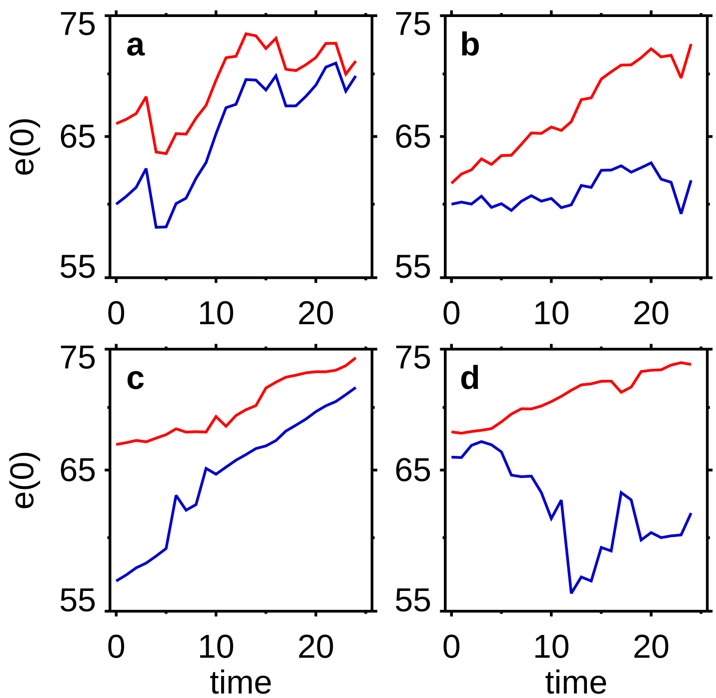

Figure 1: Examples of convergence and co-movement. Simulated e(0) trajectories for men (blue) and women (red). (a) co-movement with convergence; (b) co-movement without convergence; (c) no co-movement, but convergence; (d) neither co-movement nor convergence.

Introduction

Demographers have a long interest in mortality sex differences (Preston and Wang 2006).

The divergence of male and female mortality at the population level has also been a topic of much interest, especially in the last 10 years or so (cite Vallin, etc).

Co-movement and convergence

This paper looks at male/female co-movement of life expectancy in 40 countries around the world. We find that — even when they diverge — males and females almost always statistically co-move. We use the terms "co-movement" and "convergence" in a specific way. Hereinafter, co-movement between two time series means common signs of first differences that are statistically distinguishable from random movement. If two time series are co-moving, then when one goes up, the other one does too, and if one declines, they both decline. Statistical co-movement means that the preponderance of moves be in the same direction, not that the two time series need be in lock-step. The magnitudes of the movements do not matter, only the direction. The opposite of co-movement is anti-movement. We use the test proposed by Goodman and Grunfeld (1961) to assess co-movement. Convergence has a more intuitive definition: in a specified time window, if two time series are closer together at the end compared to the beginning, they are converging. The opposite of convergence is divergence.

Figure 1 illustrates a 2×2 lay-out of co-movement and convergence in male and female life expectancy for four simulated populations[1]. Panel (a) of figure 1 shows both co-movement and convergence. Panel (b) shows how co-movement need not imply convergence, with the two time series diverging yet co-moving. Panel (c) illustrates that convergence need not imply co-movement. Panel (d) shows divergence, with neither statistical co-movement or anti-movement. *A priori*, there is no reason male and female life expectancy in a given country cannot follow any of the combinations illustrated in figure 1.

Materials and methods

We use all available period life expectancy data from the Human Mortality Database (add date accessed). Table 2 provides basic descriptive statistics for the 40 countries. Minimum life expectancy usually occurs at, or close to, the start of the data set, and maximal values occur most recently, almost always at or near the end of the series. A prominent counterexample are Russian males, whose life expectancy peaked in 1964, as has been well documented (Shkolnikov et al. 1995; Cockerham 1999 [cite 2 more here]).

Table 3 provides descriptive statistics for life expectancy sex differentials. There is much international variation in the timing of maximum and minimum sex differences in life expectancy. The right two columns of table 3 give the 25-year window for which the divergence between male and female life expectancy is greatest. For example, for Australia, the period 1945–69 has the greatest 25-year divergence between the sexes in life expectancy: in 1945 the sex difference was 3.68 years (females, 70.35; males, 66.67) and 25 years later the difference was 6.93 (females, 74.70; males, 67.77), for a divergence

[1] These simulations use Cauchy random walks (Feller 1971).

Table 1: Goodman-Grunfeld analysis of male, female life expectancy co-movement

Sweden	entire series		25-year window	
	Females		Females	
Males	Decreases	Increases	Decreases	Increases
Decreases	87	14	3	7
Increases	9	146	0	15
p-values:				
Pearson χ^2	<0.0005		0.024	
Fisher's Exact Test	<0.0005		0.052	
Goodman-Grunfeld	<0.0005		0.063	

of 3.25 years. In this example, the end of the 25-year window does not correspond to the single-year maximal sex differential (which was 7.21 years, in 1978), nor does the start correspond to the minimal sex differential (3.39, in 1928). However, for a number of countries it is no coincidence that the window begins in the late nineteenth century, associated with extreme sex differentials a quarter-century later, during the 1914–18 world war. (make a note that only Chile has less than 25 years... and confirm that.)

The countries in the HMD are heterogeneous in the length of their data series (Sweden, the longest, has 260 years [CONFIRM]). Therefore, we perform a separate analysis, using only the data from the above-defined 25-year window for each country. Looking at 25-year subsets is one way to put all the countries on an equal footing, to ensure that differences in statistical significance are not a function of differences in sample size. We choose the 25-year subset in which there is the most divergence to plausibly give a conservative estimate of co-movement, since periods of divergence could be when there is the weakest coupling of mortality between the sexes.

We use the Goodman-Grunfeld (1961) nonparametric test of co-movement between two time series, an analysis of signs of differences cross-classified in a 2×2 table. This involves counting if male and female life expectancy both increase, or both decrease, or move in either permutation of opposite directions, on a year-to-year basis. Under the null hypothesis of no co-movement between the two data series, the expected frequencies of the four cells would be equal. The Goodman-Grunfeld test is based on the χ^2 statistic, but introduces a correction for serial correlation.

A sketch of the test follows. Take two time series $X = \{X_0, X_1, \ldots, X_n\}$ and $Y = \{Y_0, Y_1, \ldots, Y_n\}$, with subscripts for time periods. Indicator variables, $U = \{U_0, U_1, \ldots, U_{n-1}\}$ and $V = \{V_0, V_1, \ldots, V_{n-1}\}$, coded $\{0,1\}$, denote period-to-period increase (i.e., $U_i = 1$ if $X_{i+1} - X_i > 0$, etc.) Then cross-classify U and V in a 2×2 table, the cell counts of which are labeled (left to right) a, b across the top row and c, d across the bottom row; the counts of co-movements are a and d and the counter-movements are b and c (table 1 shows the layout). The Goodman-Grunfeld test statistic is:

$$\frac{a - A}{\sqrt{n[(a+b)(a+c)(b+d)(c+d)/n^4 + 2ef]}} \sim N(0,1), \quad \text{where:}$$

$$e = \sum_{i=0}^{n'-1} U_i U_{i+1}/n' - [(a+b)/n]^2, \text{ and } f = \sum_{i=0}^{n'-1} V_i V_{i+1}/n' - [(a+c)/n]^2,$$

and where $n = a + b + c + d$, $A = (a+b)(a+c)/n$, and $n' = n - 1$ is the number of sequential pairwise comparisons of U and V. The term in ef is the correction for serial correlation. The test statistic is normally distributed because of the square-root relationship between a normal and a χ^2 distribution with one degree of freedom. Although not shown in the above formula, we use the continuity correction (Yates, 1984), as Goodman and Grunfeld suggest. The Goodman-Grunfeld test is more conservative (i.e., harder to reject the null) than a naïve χ^2 analysis of the co-movements, and we display p-values from Pearson χ^2 and Fisher's 'exact' test in table 1 to illustrate this point. We use one-sided tests, since, *a priori*, there is reason to believe that male and female e(0) might co-move, but no reason to think they should anti-move. Analysis was performed using IDL 8.1 (Exelis Visual Information Solutions, Boulder, Colorado, USA).

Table 2: **Sample descriptive statistics.**

			Males				Females			
	Start	End	minimum		maximum		minimum		maximum	
Country	date	date	Value	Year	Value	Year	Value	Year	Value	Year
Australia	1921	2007	59.12	1921	79.27	2007	63.24	1921	83.79	2006
Austria	1947	2008	58.69	1947	77.62	2008	63.80	1947	82.97	2008
Belarus	1959	2007	62.19	1999	69.35	1964	71.98	1959	76.91	1974
Belgium	1841	2007	31.66	1866	76.92	2007	33.27	1866	82.34	2007
Bulgaria	1947	2007	52.54	1947	69.47	2007	55.70	1947	76.59	2007
Canada	1921	2006	55.85	1923	78.30	2006	58.05	1923	82.92	2006
Chile	1992	2005	71.46	1993	74.99	2005	77.49	1992	80.74	2005
Czech Republic	1950	2008	61.97	1950	73.98	2008	66.85	1950	80.33	2008
Denmark	1835	2007	36.65	1835	76.13	2007	39.83	1853	80.53	2007
Estonia	1959	2007	60.64	1994	67.39	2006	71.93	1959	78.76	2007
Finland	1878	2008	26.32	1918	76.33	2008	38.94	1881	83.01	2008
France	1806	2007	23.77	1813	77.43	2007	32.42	1871	84.39	2007
GDR	1956	2008	65.84	1957	76.49	2008	70.52	1957	82.36	2008
FRG	1956	2008	65.82	1957	77.55	2008	70.89	1956	82.45	2007
Hungary	1950	2006	59.85	1950	69.17	2006	64.25	1950	77.66	2006
Ireland	1950	2006	63.51	1951	77.27	2006	66.11	1951	81.89	2006
Iceland	1838	2007	16.76	1882	79.49	2003	18.82	1846	83.29	2005
Israel	1983	2007	73.10	1983	78.47	2007	76.53	1983	82.29	2007
Italy	1872	2006	23.50	1918	78.62	2006	28.33	1918	84.09	2006
Japan	1947	2008	49.78	1947	79.30	2008	53.65	1947	86.04	2008
Latvia	1959	2007	59.04	1994	67.34	1964	72.62	1994	76.72	2005
Lithuania	1959	2007	62.53	1994	68.30	1964	70.58	1959	77.63	2003
Luxemburg	1960	2006	65.42	1964	76.69	2006	71.82	1962	82.23	2004
Netherlands	1850	2006	29.88	1859	77.63	2006	31.86	1859	81.89	2006
New Zealand	1948	2003	66.97	1948	77.01	2003	70.74	1949	81.42	2003
Norway	1846	2007	43.34	1848	78.25	2007	45.78	1862	82.68	2006
Poland	1958	2006	62.43	1959	70.90	2006	68.32	1959	79.55	2006
Portugal	1940	2007	45.86	1941	75.86	2007	49.89	1941	82.19	2006
Russia	1959	2008	57.38	1994	64.89	1964	71.07	1994	74.50	1989
Slovakia	1950	2008	59.14	1950	70.82	2008	62.55	1950	78.86	2008
Slovenia	1983	2006	66.77	1983	74.40	2006	74.88	1983	81.78	2006
Spain	1908	2006	29.82	1918	77.58	2006	30.69	1918	84.07	2006
Sweden	1751	2007	17.15	1773	78.92	2007	18.79	1773	82.95	2007
Switzerland	1876	2007	38.38	1876	79.33	2007	41.52	1877	84.09	2007
Taiwan	1970	2008	66.32	1970	75.46	2008	71.42	1970	81.64	2008
England and Wales	1841	2006	33.38	1918	77.46	2006	38.14	1849	81.73	2006
Scotland	1855	2006	38.77	1864	74.84	2006	41.48	1864	79.81	2006
Northern Ireland	1922	2006	53.66	1924	76.20	2006	54.73	1925	81.15	2005
Ukraine	1959	2008	61.21	1995	68.46	1964	72.20	1959	75.19	1989
United States	1933	2006	58.34	1934	75.50	2006	62.37	1934	80.65	2006

Table 3: **Descriptive statistics, male–female difference.**

	Female−Male Difference				maximum 25-year divergence	
	Minimum		Maximum			
	value	date	value	date	magnitude	start date
Australia	3.39	1928	7.17	1980	3.25	1945
Austria	4.77	1949	7.21	1978	2.26	1949
Belarus	6.42	1962	12.21	2005	3.21	1960
Belgium	-0.47	1845	8.96	1940	4.96	1916
Bulgaria	2.84	1948	7.58	1994	3.18	1970
Canada	2.14	1924	7.39	1978	3.75	1936
Chile	5.75	2005	6.68	1996	3.75	1992
Czech Republic	4.79	1953	7.87	1990	2.32	1952
Denmark	1.32	1869	6.21	1979	3.46	1950
Estonia	7.56	1959	12.93	1995	3.83	1971
Finland	1.87	1878	23.67	1941	20.00	1917
France	0.69	1858	25.31	1915	22.49	1891
GDR	4.47	1958	7.50	1993	2.30	1970
FRG	4.88	2008	6.75	1980	1.79	1956
Hungary	3.80	1954	9.46	1994	3.98	1968
Ireland	2.23	1950	5.90	1979	2.97	1950
Iceland	-2.70	1891	10.70	1906	12.49	1867
Israel	3.43	1983	4.37	1998	12.49	1983
Italy	-0.25	1885	16.46	1917	16.19	1893
Japan	3.31	1951	6.95	2003	1.81	1951
Latvia	7.11	1959	13.58	1994	4.99	1970
Lithuania	6.04	1960	12.36	2007	4.20	1960
Luxemburg	5.07	2006	7.85	1977	2.37	1977
Netherlands	1.28	1933	10.42	1945	8.77	1921
New Zealand	3.46	1949	6.70	1973	3.24	1949
Norway	2.19	1862	6.86	1986	3.26	1949
Poland	5.76	1958	9.22	1991	2.85	1967
Portugal	4.03	1941	7.44	1992	2.24	1942
Russia	8.30	1959	13.69	1994	3.32	1970
Slovakia	3.41	1950	8.87	1990	3.79	1954
Slovenia	6.90	2005	8.25	1985	3.79	1983
Spain	0.87	1918	9.35	1937	7.42	1913
Sweden	1.24	1779	6.65	1789	3.22	1954
Switzerland	2.06	1883	7.00	1991	2.75	1894
Taiwan	4.83	1985	6.38	2006	1.53	1982
England and Wales	1.64	1849	20.02	1917	16.80	1893
Scotland	2.01	1871	7.99	1944	4.81	1918
Northern Ireland	0.21	1927	7.14	1974	4.31	1937
Ukraine	6.37	1960	11.90	2007	3.35	1972
United States	3.65	1933	7.73	1975	2.73	1939

Results and discussion

Table 4 presents the results of the Goodman-Grunfeld testing. When using the entire time series for each country, only two countries, do not have significant co-movement of life expectancy between the sexes. These are Chile, which has only 14 years of data, and Luxembourg, which has ample data but appears to be an outlier. All the other G-G test statistics amply reject the null hypothesis of no co-movement at the 5% level.

Conclusion

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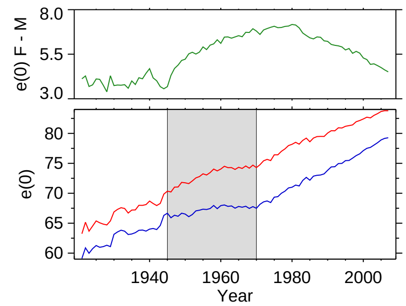
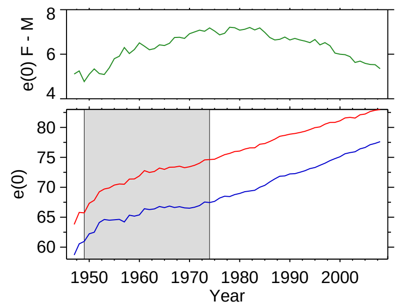
Table 4: **Goodman-Grunfeld test statistics and p-values.**

	Entire Series		25-year Window	
	test statistic	one-sided p-value	test statistic	one-sided p-value
Australia	5.473	<.00005	3.468	0.0003
Austria	2.875	0.0020	2.239	0.0126
Belarus	3.935	<.00005	2.142	0.0161
Belgium	9.658	<.00005	2.820	0.0024
Bulgaria	5.649	<.00005	2.097	0.0180
Canada	5.861	<.00005	1.418	0.0781
Chile	-0.595	0.7242	-0.595	0.7242
Czech Republic	5.704	<.00005	2.576	0.0050
Denmark	8.598	<.00005	1.073	0.1416
Estonia	2.850	0.0022	1.593	0.0556
Finland	7.747	<.00005	2.973	0.0015
France	11.482	<.00005	4.089	<.00005
GDR	4.757	<.00005	3.229	0.0006
FRG	3.810	0.0001	3.046	0.0012
Hungary	3.928	<.00005	2.465	0.0069
Ireland	3.024	0.0012	2.723	0.0032
Iceland	5.186	<.00005	3.749	0.0001
Israel	3.250	0.0006	3.250	0.0006
Italy	9.304	<.00005	3.172	0.0008
Japan	4.982	<.00005	2.639	0.0042
Latvia	2.346	0.0095	1.496	0.0673
Lithuania	2.956	0.0016	1.614	0.0532
Luxemburg	-0.273	0.6076	-0.429	0.6660
Netherlands	8.707	<.00005	3.702	0.0001
New Zealand	2.300	0.0107	2.026	0.0214
Norway	7.811	<.00005	1.753	0.0398
Poland	4.390	<.00005	2.328	0.0100
Portugal	5.642	<.00005	4.349	<.00005
Russia	4.562	<.00005	2.781	0.0027
Slovakia	2.977	0.0015	2.709	0.0034
Slovenia	1.779	0.0376	1.779	0.0376
Spain	7.114	<.00005	3.215	0.0007
Sweden	12.863	<.00005	1.512	0.0653
Switzerland	7.144	<.00005	4.322	<.00005
Taiwan	2.358	0.0092	1.906	0.0283
England and Wales	10.794	<.00005	3.400	0.0003
Scotland	7.999	<.00005	3.373	0.0004
Northern Ireland	4.408	<.00005	2.184	0.0145
Ukraine	4.016	<.00005	1.671	0.0473
United States	6.221	<.00005	4.182	<.00005



Australia



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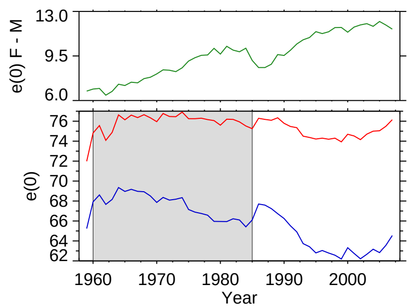

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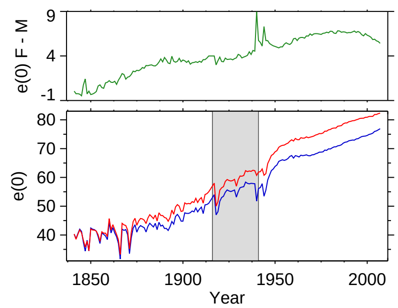

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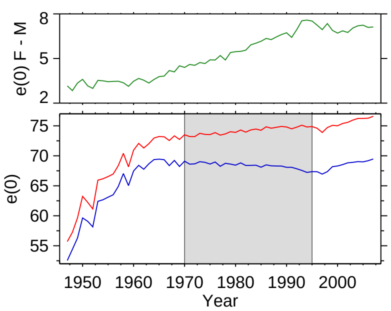

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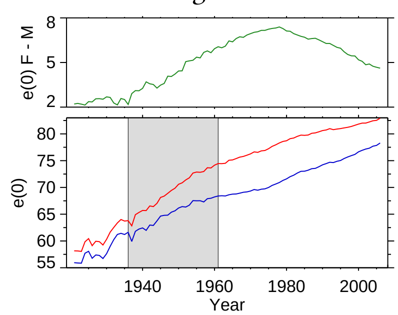

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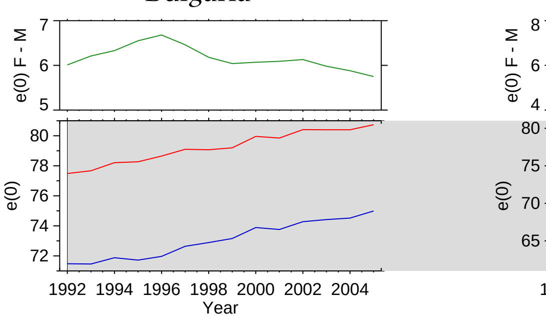

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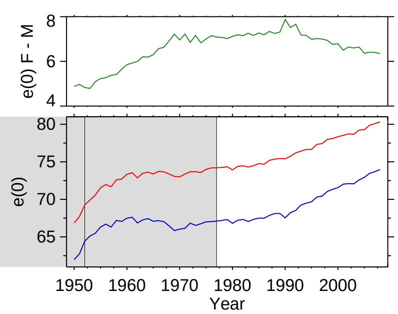

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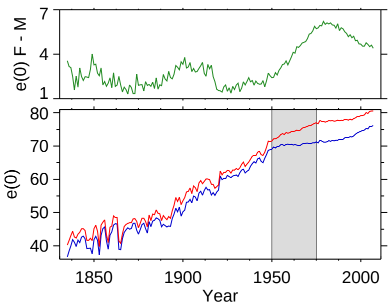

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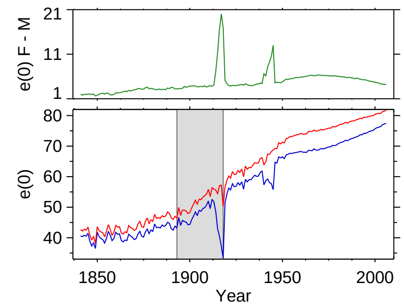
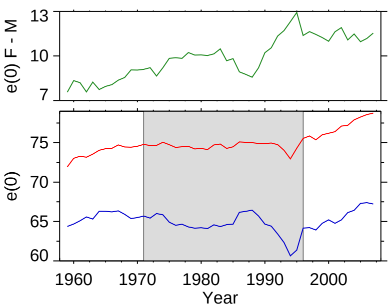


England & Wales



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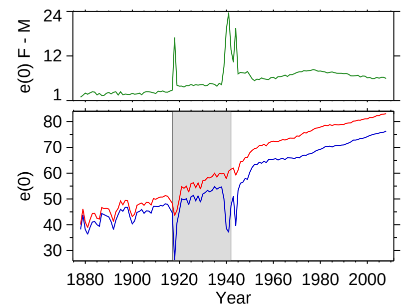

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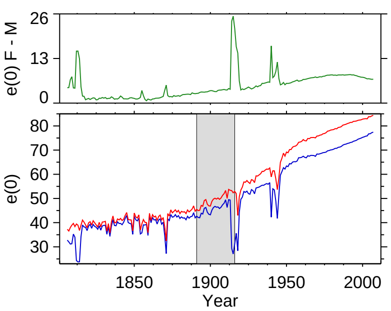

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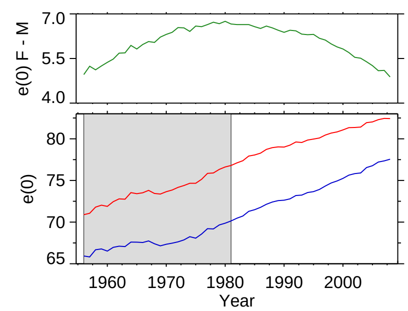
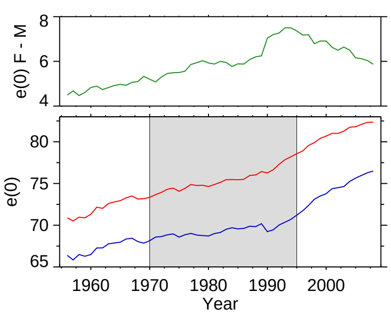


Germany (West))



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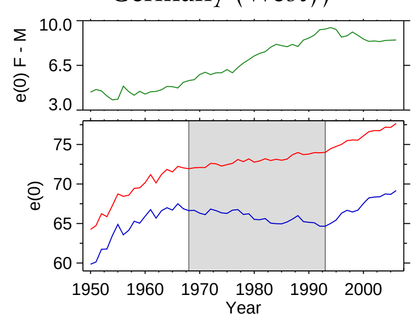

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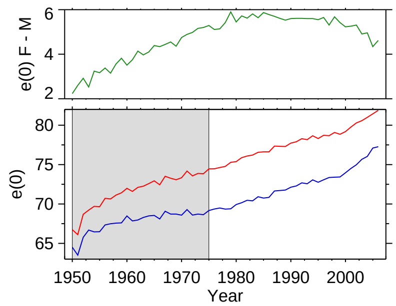

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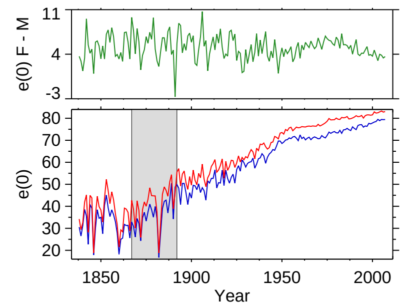

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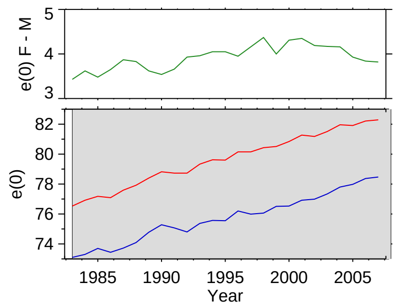
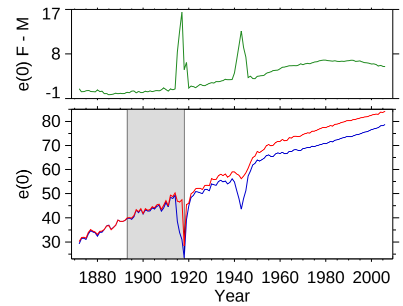


Israel



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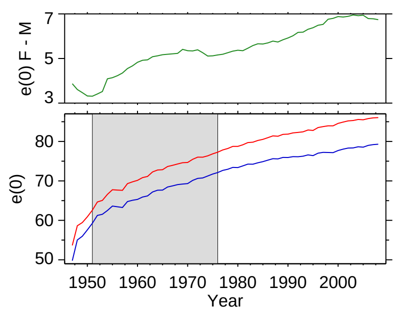

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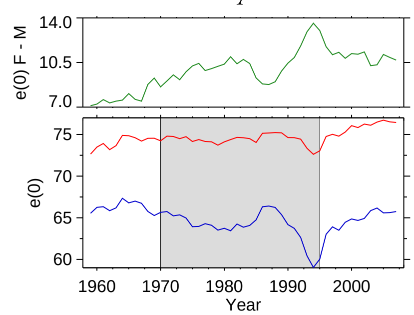

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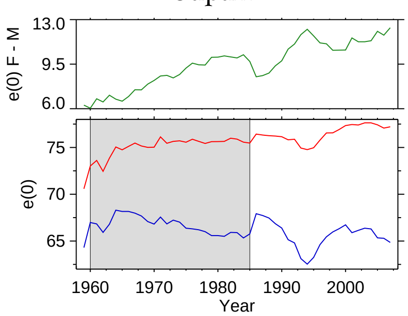

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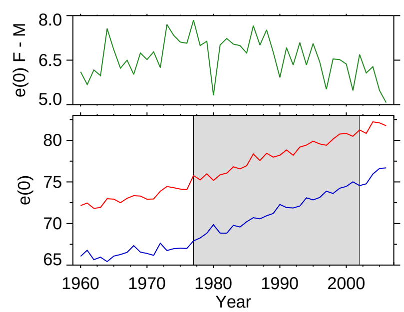

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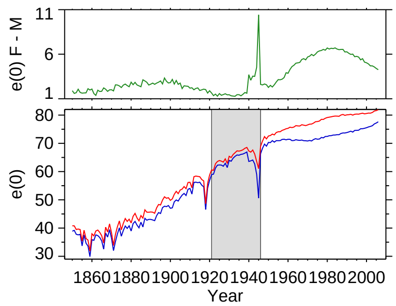

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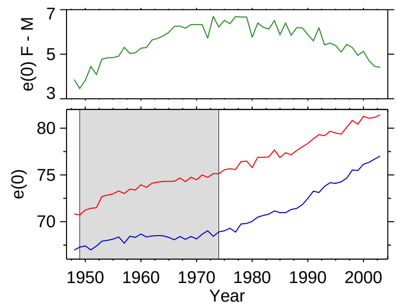

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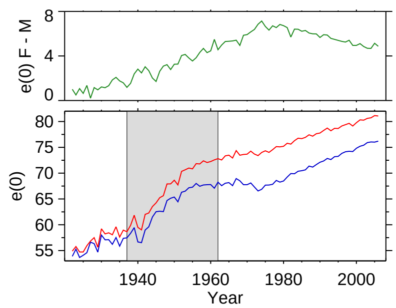

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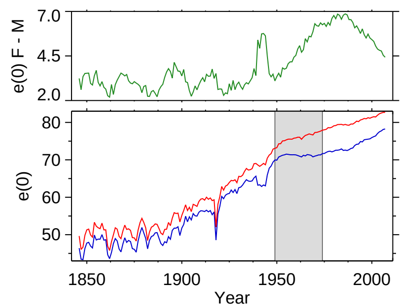

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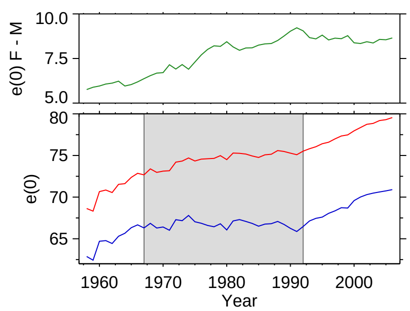

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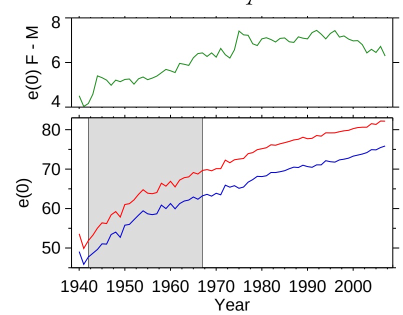

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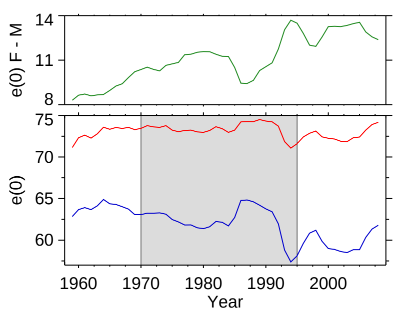

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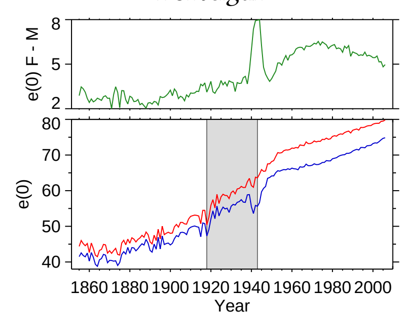

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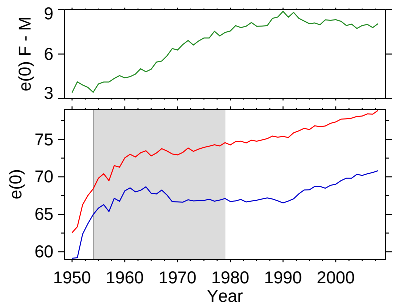

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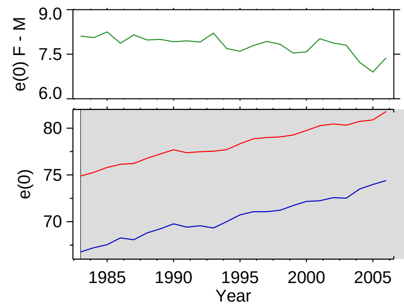

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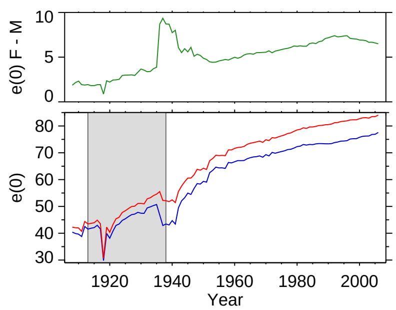

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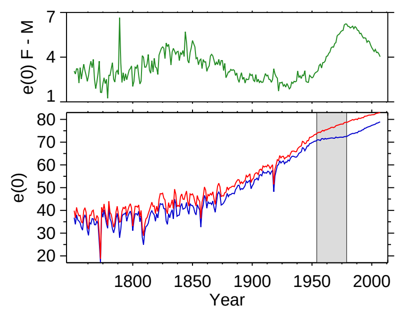

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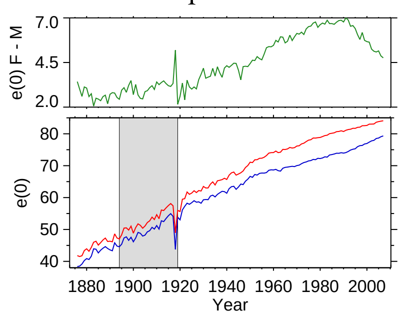

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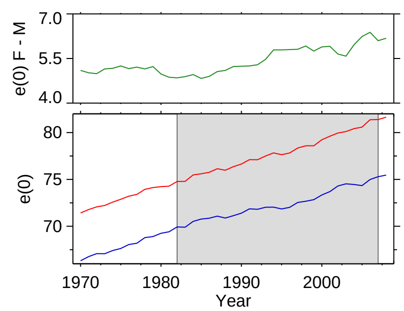

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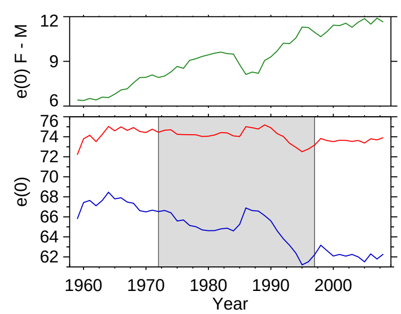

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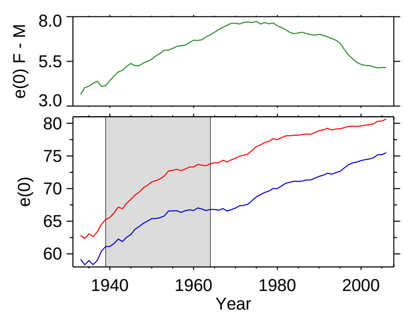

United States