

1 **Mortality change and lifespan inequality**

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23

1 In the past six decades, lifespan inequality has varied greatly within and among
2 countries even while life expectancy has continued to increase. How and why does
3 mortality change generate this diversity? We derive a precise link between changes
4 in age-specific mortality and lifespan inequality, measured as the variance of age at
5 death. Key to this relationship is a young–old threshold age, below and above which
6 mortality decline respectively decreases and increases lifespan inequality. First, we
7 show that the threshold’s location modifies the correlation between changes in life
8 expectancy and lifespan inequality, illustrating this over two centuries in Sweden.
9 Second, we analyze the post Second World War trajectories of lifespan inequality in
10 a set of developed countries, focusing on Japan, Canada and the United States (US).
11 Here, the thresholds centered on retirement age. Our analysis shows that the focus
12 of Japanese mortality decline shifted gradually from young to old, causing lifespan
13 inequality to switch from decrease to modest increase ca. 1990. Early in the 1980s,
14 mortality increases in young US males, led lifespan inequality to remain high in the
15 US, while in Canada there were steady declines. In general, post Second World War
16 mortality change varied most at young working ages, particularly in males. We
17 conclude that if mortality decline continues to stagnate at young ages, but to
18 progress steadily at old ages, lifespan inequality increases will become more
19 common.

20

21

1 **Introduction**

2 Lifespan inequality is the defining measure of social and health disparity and, alongside
3 life expectancy (the mean age at death), is a key indicator of population health
4 (Tuljapurkar et al. 2000; Edwards and Tuljapurkar 2005). Changes in life expectancy are
5 well understood in terms of the underlying changes in mortality: mortality reduction at
6 any age increases life expectancy (Keyfitz 1977; Goldman and Lord 1986; Vaupel 1986;
7 Vaupel and Canudas-Romo 2003). However, the precise link between lifespan inequality
8 and age-specific mortality is less clear. First, only mortality decline at ages below a
9 young–old threshold can decrease lifespan inequality (Zhang and Vaupel 2009; Vaupel et
10 al. 2011; van Raalte and Caswell 2012). Second, there are several ways to quantify
11 lifespan inequality and understand its change through time (Wilmoth and Horiuchi 1999;
12 Cheung et al. 2005; Edwards and Tuljapurkar 2005; Horiuchi et al. 2008; Vaupel et al.
13 2011). Although highly correlated, these measures behave differently in response to
14 change in the age structure of mortality (van Raalte and Caswell 2012). Here, we measure
15 lifespan inequality by the variance of age at death, which measures the dispersion in age
16 at death relative to life expectancy (Edwards and Tuljapurkar 2005). The variance of age
17 at death is a central parameter in population and economic modeling (Tuljapurkar 2008;
18 Caswell 2009; Tuljapurkar and Edwards 2011; Edwards 2012; Schindler et al. 2012) and
19 it is therefore important to understand how it responds to mortality change. We present an
20 exact relationship between age-specific mortality and the variance of age at death—a
21 more transparent and demographically interpretable relationship than that derived by
22 Caswell (Eq. 42 in Caswell (2009)). Our framework provides a powerful tool for

1 quantifying how trajectories of age-specific mortality affect the variance, both within
2 countries and in an international sample of countries.

3

4 Given period age-specific mortality, $\mu(x)$, and the corresponding probability of survival to
5 each age $l(x)$, the age-distribution of death is $\varphi(x) = \mu(x)l(x)$. Figure 1 contrasts the age-
6 distribution of death in Sweden in 1800 and 2000, illustrating the standard outcomes of
7 development: (i) reduction of child death; (ii) increase of life expectancy; (iii) emergence
8 and advance of a modal age at adult death; (iv) decrease of lifespan inequality (Lee and
9 Carter 1992; Bongaarts 2005; Canudas-Romo 2008; Vaupel et al. 2011). These arise from
10 an initial focus of mortality decline on children and young adults followed by a shift of
11 mortality decline to older ages (Cutler et al. 2006).

12

13 Since about 1950 in the developed countries, life expectancy has increased consistently
14 but change in lifespan inequality among countries has varied greatly (Edwards and
15 Tuljapurkar 2005; Peltzman 2009; Smits and Monden 2009; Engelman et al. 2010). This
16 is in striking contrast to the strong negative correlation between changes in life
17 expectancy and lifespan inequality present during the first century of mortality decline
18 (Vaupel et al. 2011). The recent variability of this relationship, we show, results from the
19 very different responses of life expectancy and lifespan inequality to age-specific
20 mortality change. First, we show that in Sweden, the threshold for the variance of age at
21 death increased from childhood in the late 18th century to around 65 years (y) in 1950.
22 This movement of the threshold altered the relationship between changes in life
23 expectancy and lifespan inequality. Second, we examine the pattern of post Second

1 World War lifespan inequality across many countries, and then focus on Japan, Canada
2 and the United States (US). The application of our decomposition to these post war
3 trajectories of lifespan inequality showed that fluctuations in lifespan inequality resulted
4 primarily from mortality change at young working ages.

5

6 **Results**

7 **The analysis of change**

8 The variance of age at death after a specified index age A is

$$9 \quad V(A) = \frac{1}{l(A)} \int_A^{\infty} \varphi(x) [x - e(A)]^2 dx, \quad (1)$$

10 where $e(A)$ is the life expectancy after age A . If we set $A=0$, life expectancy is $e(0)$ and
11 the variance, $V(0)$, quantifies the dispersion among deaths at all ages. However, for
12 developed countries it can be more informative to focus on adult ages, e.g. using $V(15)$ ¹.

¹ $V(0)$ is necessarily dominated by the (now usually small) fraction of infant deaths. If we set $A=15$, life expectancy $e(15)$ and the variance $V(15)$ describe the dispersion of adult deaths. The advantage of focusing on adult ages is that the effects of infant mortality change are removed. Infant mortality has fallen steadily, driving steady declines in $V(0)$ (Edwards and Tuljapurkar 2005; Tuljapurkar and Edwards 2011). Considering only mortality after particular index ages A can often reveal quite different trajectories of $V(A)$ (Engelman et al. 2010). Even with low infant mortality, the inclusion of the few youngest ages can obscure the lifespan inequality effects of adult mortality change. This makes $V(15)$ an ideal measure for investigating how the variability in age-specific mortality trajectories contributes to lifespan inequality change.

1

2 What is the change in $V(A)$ due to a proportional mortality reduction at an age x greater
3 than A ? Starting from Eq. 1, we find

$$4 \quad \frac{dV(A)}{d \ln \mu(x)} = \frac{-2\mu(x)}{l(A)} \int_x^\infty l(z)[z - e(A)]dz, \quad (2)$$

5 (for details, see Supplementary Information). The integral in Eq. 2 has a negative term:
6 the contribution from ages between x and $e(A)$, and a positive term: the contribution from
7 ages above $e(A)$. If mortality decreases at an age above $e(A)$, the variance $V(A)$ will
8 increase. Now suppose that we reduce mortality at an age below $e(A)$. It is clear from Eq.
9 2 that reducing mortality at younger ages will gradually cause the negative contribution to
10 dominate, thus decreasing $V(A)$. We therefore define a young–old threshold age, which
11 we call $T(A)$, before which mortality decline decreases $V(A)$, and after which mortality
12 decline increases $V(A)$.

13

14 Hereafter, we refer to the rate of change in Eq. 2 as the sensitivity of $V(A)$. Figure 2a
15 shows the sensitivity of the variance for $A=15$, for Sweden in 1950 and 2011. Three
16 features of the plot are typical of mortality in developed countries: (i) mortality decline at
17 ages from 15 to about 65 y decreases V ; (ii) mortality decline after about 65 y increases
18 V ; (iii) there is a threshold, $T(15)+15$, at approximately age 65 y, which separates ‘young’
19 ages, where mortality decline decreases V , from ‘old’ ages, where mortality decline
20 increases V . The sensitivity of life expectancy $e(15)$ to a proportional mortality decline at
21 age x (Goldman and Lord 1986; Vaupel 1986; Vaupel and Canudas-Romo 2003) is
22 shown for comparison in Figure 2b. The obvious distinctions are that mortality decline at
23 any age increases life expectancy; and that mortality decline close to the threshold has

1 least effect on V . The great difference in the shape of these sensitivities of life expectancy
2 and lifespan inequality, especially below the threshold age, is the critical factor driving
3 their divergent responses to the same age structure of mortality change.

4

5 **Long-term mortality change**

6 Sweden was the first country to record ages at death systematically, with records from
7 1751 to the present (Human Mortality Database 2012). These data provide the best
8 opportunity to understand the long-term association of the young–old threshold with
9 changes in lifespan inequality. Figure 3 displays the annual change in the thresholds
10 $T(A)+A$, for index ages $A=0, 15$, and 65 y. Figure 4 shows the concurrent changes in $e(A)$
11 and $V(A)$.

12

13 To provide some basic intuition on how the threshold depends on the age structure of
14 mortality, we note that a lower bound on the threshold for $A=0$ (derived from Eq. 2 in the
15 Supplementary Information) is

$$16 \quad T(0) \geq 2e(0) - \omega, \quad (3)$$

17 where ω is the longest lifespan. In the early years of transition to low mortality, before
18 the late 19th century, the lower bound defined by Eq. 3 was below or close to zero. In the
19 first half of the 20th century, life expectancy increased rapidly, which substantially
20 increased the threshold age—e.g. with $e(0)=65$ and $\omega=100$ we have a lower bound at 30
21 y. More recently, life expectancy has begun to converge on ω . This convergence is
22 commonly known as ‘rectangularization’ of the age structure (Wilmoth and Horiuchi

1 1999), and it causes the lower bound on the threshold to advance. We now turn to exact
2 values of the young–old threshold.

3
4 Before about 1875, the threshold $T(0)$ followed a fluctuating increase to about 20 y (Fig.
5 3), so only mortality decline in children and very young adults could decrease V .
6 However, $V(0)$ changed little (Fig. 4a), indicating an even distribution of mortality
7 decline below and above the threshold age. From 1875 to about 1950, the threshold
8 advanced to near 60 y, so mortality decline at a greater range of young ages could now
9 decrease V . This reinforced the strong negative correlation between changes to life
10 expectancy and lifespan inequality. In this period, V decreased rapidly and life
11 expectancy increase accelerated, indicating much faster mortality declines below the
12 threshold age. After 1950, the threshold continued advancing to 70–75 y. In this recent
13 period, decreases in V slowed, despite continued increases in life expectancy, indicating
14 the switch to more even mortality decline below and above the threshold age.

15
16 Increasing the index age after which we measure lifespan inequality advances the young-
17 old threshold. To investigate changes in lifespan inequality among adults, we use an
18 index of 15 y. $V(15)$ did not begin to decrease rapidly until ca. 1925 (Fig. 4b), when the
19 threshold $T(15)+15$ was 57 y (Fig. 3). That this decrease began around 50 y later than the
20 decrease in $V(0)$ indicates the much later spread of rapid mortality decline from below 15
21 y to 15–57 y. By 1950, mortality reduction below 15 y had converged the thresholds $T(0)$
22 and $T(15)+15$ to 60–65 y. This initiated a period when the distribution of mortality

1 change between working and retired ages was critical in shaping the trajectory of lifespan
2 inequality.

3

4 Consider now an index age of 65 y: the obvious feature of mortality change is the
5 consistent positive relationship of changes in life expectancy and lifespan inequality (Fig.
6 4c). The threshold $T(65)+65$ was always near 70 y, so only mortality decline at a few
7 young ages could decrease $V(65)$. Therefore, even with mortality reduction by an equal
8 proportion at each age, there was a strong bias for variance increase. The only deviation
9 occurred after 1980, when the trajectory of $V(65)$ flattened while $e(65)$ increased; the
10 threshold was 75 y in 1980. Comparison of mortality change at ages 65–75 y, and >75 y
11 showed an abrupt increase in mortality decline among males, which was greatest at 65–
12 75 y (Fig. S1). This was large enough to overwhelm the bias for variance increase and,
13 for the first time since 1751, disrupted the positive correlation between increases in life
14 expectancy and lifespan inequality. We now return to lifespan inequality across the full
15 range of adult ages.

16

17 **The young–old threshold for mortality in developed countries**

18 In developed countries the age-distribution of death has a well-defined modal age in
19 adulthood (Fig. 1). This makes it a reasonable fit to the Gompertz model in which
20 mortality increases with age as

$$21 \quad \mu(x) = Be^{kx}, \quad (4)$$

1 where B is the initial level of mortality and k is the exponential rate of mortality increase.
2 From Eq. 4 (see Supplementary Information and Pollard (1991)) the mode (m) and
3 standard deviation (σ) of age at death are

$$4 \quad m = \frac{1}{k} \ln \frac{k}{B}, \quad (5)$$

5 and

$$6 \quad \sigma = \frac{1}{k}. \quad (6)$$

7 Life expectancy is given to a good approximation by

$$8 \quad e(0) \cong m - \frac{\sigma}{2}, \quad (7)$$

9 and our threshold is approximated by

$$10 \quad T(0) \cong m - \frac{3\sigma}{2}. \quad (8)$$

11 This places the threshold at 1.5 standard deviations of the age at death below the modal
12 age, and (using Eq. 7) 1 standard deviation below life expectancy. The threshold's
13 advance is therefore driven by increases in both adult life expectancy and modal age at
14 death.

15

16 **Age-specific drivers of lifespan inequality change in developed countries post-1947**

17 We now focus on the post war changes to $V(15)$, for which the young–old threshold was
18 largely at ages 60–70 y. Our analysis shows that changes in lifespan inequality were
19 shaped by striking differences in mortality change for young versus old adults. Figure 5
20 (solid lines) shows the average sex-specific trajectories of $V(15)$ across 40 regional data
21 series in the Human Mortality Database (Vaupel et al. 2011). The average variance

1 trajectory fluctuated more for males, and the annual direction of variance change was less
2 consistent among regions for males (Fig. S2). In Figure 5, we also show (dashed lines)
3 the variance effects of mortality change at young ages (below the threshold) and at old
4 ages (above the threshold), with each component again averaged across regions.
5 Fluctuations in the trajectories of variance change were clearly driven by mortality
6 change at young ages, particularly in males. This conclusion is supported by the pattern
7 (grey shading) of the annual 95% range among regions of the young and old components
8 of variance change.

9

10 We now examine in more detail Japan, Canada and the United States (US), which
11 illustrate the main features of post-1947 mortality change in developed countries:
12 relatively steady increase in life expectancy (Fig. S3), but large fluctuation in V (Fig. S4).
13 We focus on two features: the rapid decrease in variance immediately after 1947, which
14 was especially pronounced in Japan; and the slowing of variance decrease or onset of
15 variance increase in the 1980s. The US in particular showed a striking variance increase
16 in the early 1980s. The average threshold age was close to the conventional age of
17 retirement, although there was a steady increase in the thresholds over time—by a
18 maximum of 28 y in Japanese females (Fig. S5). Figure 6 shows how mortality change at
19 young ages (below the threshold) and at old ages (above the threshold) generated the
20 contrasting variance trajectories of each country. We also computed the components of
21 variance change from below and above the fixed age of 65 y (Fig. S6). These components
22 were near identical to those either side of the shifting threshold, indicating that mortality
23 change near the threshold had minimal influence. We now focus on Fig. 6, which

1 presents the components of variance change from below and above the shifting threshold
2 age.

3

4 *Japan*

5 Each sex in Japan followed the same basic trajectory of variance change: a rapid decrease
6 from 1947 to about 1975, and then increase after about 1990 (Fig. 6a & b). The sexes
7 differed in that V initially decreased faster for females, and the reversal from variance
8 decrease to increase was earlier for females. Our decomposition (considering successive
9 periods of ten-years) showed that variance decrease before 1977 was driven by the
10 variance contracting effects of mortality decline at young ages, which dominated the
11 effects of mortality decline at old ages (Fig. 6a & b). From 1977 to 1987, these opposing
12 effects reached a rough equilibrium and so V changed little. After 1987, the negative
13 effects of mortality decline at young ages weakened further, which led the positive effects
14 of mortality decline at old ages to dominate. This shift in the focus of mortality decline to
15 old ages led V to increase (Fig. S7). Females had an initially faster variance decrease
16 because they had faster mortality declines at young-ages (Fig. S7). Females switched
17 earlier from variance decrease to increase because, after 1987, male mortality decline
18 slowed at old ages leaving females the larger positive component of variance change.
19 After 1997, the continued slowing of mortality decline at young ages caused variance
20 increase for both females and males.

21

22 *Canada and the US*

1 In Canada and the US the variance changes after 1947 were much smaller than in Japan
2 (note the difference in y-axis scales in Fig. 6). Prior to 1983, Canada and the US followed
3 similar trajectories, each with little net variance change. However, after 1983, V increased
4 and continued to fluctuate in the US, but began a rapid and consistent decrease in Canada.
5 The result was that, from 1983 to 2007, the Canadian variance (for females and males
6 respectively) change from 8% and 9% lower than the US, to 20% and 15% lower.

7

8 We now focus on the period 1983–1994, which captures the major variance divergence
9 between Canada and the US (Fig. 6 c–f). In Canada, mortality decline at young ages
10 produced percentage variance changes of –10.8% for females and –10.1% for males (Fig.
11 6c & d). From old ages the changes were 1.0% in females and 2.9% in males. In the US,
12 mortality change at young ages produced variance changes of –3.3% in females and 3.6%
13 in males, and from old ages 1.5% and 4.8% respectively (Fig. 6e & f). Thus, due mainly
14 to differences in male mortality change, the US produced a larger component of variance
15 increase from old ages, and a much smaller component of variance decrease (and for
16 males, variance increase) from young ages.

17

18 How did mortality change differ at young ages between Canada and the US? In the US,
19 male mortality increased at ages 15–22 and 27–45 y, with increases greater than 30% at
20 34–37 y (Fig. S8). US females showed a similar pattern, but of smaller magnitude (a
21 maximum increase of 20% at 35 y). By contrast, mortality increase for Canadian males
22 was limited to ages 32–40 y, with a maximum increase of 17%; mortality generally
23 declined for Canadian females. The age-specific changes in mortality therefore differed

1 most between Canada and the US at young working ages, particularly in males. The
2 resulting sharp variance divergence between Canada and the US is in stark contrast to
3 their very similar trajectories of life expectancy during this period (Fig. S3).

4

5 **Discussion**

6 We have shown how change in lifespan inequality, as measured by V , depends strongly
7 on the balance of mortality change around a young–old threshold. The threshold advances
8 with increases in life expectancy, the modal age at death, and with the compression of
9 mortality into a narrower range of ages. Before about 1950, changes to life expectancy
10 and lifespan inequality had a strong negative correlation (Smits and Monden 2009;
11 Vaupel et al. 2011). This arose because the emphasis of mortality decline was on ages
12 below the threshold (Vaupel et al. 2011). Since 1950, lifespan inequality decrease has
13 slowed, and occasionally even reversed. We show that the post Second World War
14 fluctuations lifespan inequality were driven by fluctuations in mortality at young working
15 ages, below the threshold age, while at old ages mortality decline was relatively
16 consistent. The implication of these trends is clear: if mortality at young ages continues to
17 fluctuate, and mortality at old ages continues its steady decline, lifespan inequality
18 increases will become more common.

19

20 But what drives these trajectories of age-specific mortality, causing them to vary both
21 within and among countries? Age-specific mortality is ultimately associated with
22 education, income, social support (from family or public institutions, including the
23 provision of healthcare), lifestyle, disease and living conditions. These interact with the

1 proximate, health related, causes of death to generate lifespan inequality (Edwards and
2 Tuljapurkar 2005; Shkolnikov et al. 2011; Nau and Firebaugh 2012). For example,
3 Japanese development after 1947, including the start of universal healthcare in 1961,
4 generated rapid mortality decline at mainly working ages. However, social gradients in
5 age-specific health and mortality were also strengthening, characterized widening
6 differences in age-specific mortality by education and income, particularly for young
7 adult males (Fukuda et al. 2004; Kagamimori et al. 2009). At the same time, Japan's
8 rapid progress against young-age mortality likely reduced the scope for further mortality
9 decline at young ages, by leaving only the harder to prevent causes of death.

10

11 A great deal of research is aimed at understanding why young adult mortality remains
12 high in the US compared to other high-income countries (Crimmins et al. 2011; IOM
13 2012). One source may be high geographic disparity in young adult mortality rates within
14 the US. For example, Cullen et al. (2012) showed that mortality differences below age 70
15 among US counties resulted largely from differences in education and income,
16 particularly for males (see also Backlund et al. 2007; Crimmins et al. 2009). Recent
17 evidence also suggests that life expectancy differences between educational groups in the
18 US have widened in recent decades (Olshansky et al. 2012). A part of this variation is
19 likely due to differences in healthcare access among young adults (Crimmins et al. 2011).
20 In the US, healthcare access at working ages depends largely on employer-provided
21 health insurance; in 1994, 18.6% of non-elderly adults in the US had no insurance
22 (Holahan and Kim 2000). Being without insurance limits care, particularly preventive
23 care at young ages (Lasser et al. 2006), with evidence for consequently higher mortality

1 (Wilper et al. 2009). The 1980s lifespan inequality divergence between Canada and the
2 US is likely explained by such social factors, that likely interact with the common causes
3 of young adult death—drugs, violence and infectious disease (Nau and Firebaugh 2012).
4 For example, HIV/AIDS became prevalent in the 1980s and early 1990s, particularly
5 among US males aged 25–64 y (Armstrong et al. 1999; UNAIDS 2013).

6

7 *Alternate inequality metrics*

8 There are several established metrics of lifespan inequality (van Raalte and Caswell
9 2012). For the same mortality data, the young–old threshold can differ widely among
10 these metrics, e.g. by around 20 y between V and the metric known as lifespan disparity
11 or e^\dagger (Zhang and Vaupel 2009). Our results indicate that the major post Second World
12 War mortality changes occurred outside the zone of threshold disagreement among
13 metrics, indicating that our qualitative conclusions are robust to the choice of metric. We
14 use V for several reasons: it is easily understood and interpreted as a measure of relative
15 dispersion, easily decomposed (Edwards and Tuljapurkar 2005; Edwards 2011; Nau and
16 Firebaugh 2012), and finds direct use in analyzing the demographic and economic
17 consequences of mortality change (Tuljapurkar 2008; Edwards 2012; Schindler et al.
18 2012).

19

20 *The future*

21 The young–old threshold in developed countries is now approaching 75–80 y (Figure 2a),
22 so only mortality decline at the oldest retired ages can increase lifespan inequality. Given
23 the ongoing fluctuation of age-specific social gradients in health and mortality, and the

1 importance of social mortality gradients for generating lifespan inequality (Cullen et al.
 2 2012; Olshansky et al. 2012), we suggest that future work focus on the population-level
 3 effects of mortality change within specific subgroups (Edwards 2011; Shkolnikov et al.
 4 2011; van Raalte et al. 2011; Nau and Firebaugh 2012; van Raalte et al. 2012). Our
 5 results highlight that without an increased consistency of young-adult mortality decline,
 6 we cannot assume that future mortality change will follow the historical pattern of
 7 increasing life expectancy and decreasing lifespan inequality.

8

9 **Materials and Methods**

10 We downloaded our data from the Human Mortality Database in January 2013 (Human
 11 Mortality Database 2012), see the Background Information for each country,
 12 www.mortality.org. For international comparisons, we used the same data series as
 13 Vaupel et al. (2011). Table S1 shows the range of years between 1947 and 2011 covered
 14 by these data. We conducted all calculations in the R environment (R Development Core
 15 Team 2012). The Supplementary Information gives the derivation of each formula
 16 presented. For our age-decomposition of change in $V(15)$, we adapted Eq. 2 to

$$17 \quad \frac{dV(15)}{dt} = -2 \int_{15}^{\infty} \frac{a(x)\mu(x)}{l(15)} \int_x^{\infty} l(z)[z - e(15)] dz dx, \quad (9)$$

18 which gives the change in $V(15)$ over time interval dt . The observed proportional changes
 19 in age-specific mortality, $a(x)$, between years t and $t+1$ were computed as

$$20 \quad a(x) = \frac{\mu(x, t+1) - \mu(x, t)}{\mu(x, t)}. \quad (10)$$

21 To convert time in these formulae to discrete 1 y intervals, we discretized the probability
 22 density function of age at death, considered survivorship to the mid-point of each age-

1 interval, and substituted instantaneous mortality for central death rates. We counted age
2 in 1 y intervals starting from 0.5 y. Where we present mortality change over wider age
3 intervals, we compute a weighted average of the 1 y central death rates in each interval,
4 using the probability of survival to each age as weights (Ahmad et al. 2001).

5

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9

1 **References**

- 2 Ahmad, O., C. Boschi-Pinto, A.D. Lopez, C.J.L. Murray, R. Lozano, and M. Inoue. 2001.
3 "Age standardization of rates: a new WHO standard." Geneva: World Health
4 Organization.
- 5 Armstrong, G.L., L.A. Conn, and R.W. Pinner. 1999. "Trends in infectious disease
6 mortality in the United States during the 20th century." *Jama-Journal of the American*
7 *Medical Association* 281(1):61-66.
- 8 Backlund, E., G. Rowe, J. Lynch, M.C. Wolfson, G.A. Kaplan, and P.D. Sorlie. 2007.
9 "Income inequality and mortality: A multilevel prospective study of 521248 individuals
10 in 50 US states." *International Journal of Epidemiology* 36(3):590-596.
- 11 Bongaarts, J. 2005. "Long-range trends in adult mortality: Models and projection
12 methods." *Demography* 42(1):23-49.
- 13 Canudas-Romo, V. 2008. "The modal age at death and the shifting mortality hypothesis."
14 *Demographic Research* 19:1179-1204.
- 15 Caswell, H. 2009. "Stage, age and individual stochasticity in demography." *Oikos*
16 118(12):1763-1782.
- 17 Cheung, S.L.K., J.M. Robine, E.J.C. Tu, and G. Caselli. 2005. "Three dimensions of the
18 survival curve: Horizontalization, verticalization, and longevity extension." *Demography*
19 42(2):243-258.
- 20 Crimmins, E.M., J.K. Kim, and T.E. Seeman. 2009. "Poverty and biological risk: The
21 earlier "aging" of the poor." *Journals Of Gerontology Series A-Biological Sciences And*
22 *Medical Sciences* 64(2):286-292.

- 1 Crimmins, E.M., S.H. Preston, and B. Cohen. 2011. "Explaining divergent levels of
2 longevity in high-income countries." Panel on understanding divergent trends in
3 longevity in high-income countries; National Research Council.
- 4 Cullen, M.R., C. Cummins, and V.R. Fuchs. 2012. "Geographic and racial variation in
5 premature mortality in the U.S.: Analyzing the disparities." *PLOS one* 7(4):e32930.
- 6 Cutler, D., A. Deaton, and A. Lleras-Muney. 2006. "The determinants of mortality."
7 *Journal of Economic Perspectives* 20(3):97-120.
- 8 Edwards, R.D. 2011. "Changes in world inequality in length of life: 1970-2000."
9 *Population And Development Review* 37(3):499-528.
- 10 Edwards, R.D. 2012. "The cost of uncertain life span." *Journal of Population*
11 *Economics*:1-38.
- 12 Edwards, R.D. and S. Tuljapurkar. 2005. "Inequality in life spans and a new perspective
13 on mortality convergence across industrialized countries." *Population And Development*
14 *Review* 31(4):645-674.
- 15 Engelman, M., V. Canudas-Romo, and E.M. Agree. 2010. "The implications of increased
16 survivorship for mortality variation in aging populations." *Population And Development*
17 *Review* 36(3):511-539.
- 18 Fukuda, Y., K. Nakamura, and T. Takano. 2004. "Municipal socioeconomic status and
19 mortality in Japan: sex and age differences, and trends in 1973-1998." *Social Science &*
20 *Medicine* 59(12):2435-2445.
- 21 Goldman, N. and G. Lord. 1986. "A new look at entropy and the life table." *Demography*
22 23(2):275-282.

1 Holahan, J. and J. Kim. 2000. "Why does the number of uninsured Americans continue to
2 grow?" *Health Affairs* 19(4):188-196.

3 Horiuchi, S., J.R. Wilmoth, and S.D. Pletcher. 2008. "A decomposition method based on
4 a model of continuous change." *Demography* 45(4):785-801.

5 Human Mortality Database. 2012. "University of California, Berkeley (USA), and Max
6 Planck Institute for Demographic Research (Germany). Available at www.mortality.org
7 or www.humanmortality.de."

8 IOM. 2012. *How far have we come in reducing health disparities?: Progress since 2000:*
9 *Institute of Medicine workshop summary*. Washington, DC: The National Academies
10 Press.

11 Kagamimori, S., A. Gaina, and A. Nasermoaddeli. 2009. "Socioeconomic status and
12 health in the Japanese population." *Social Science & Medicine* 68(12):2152-2160.

13 Keyfitz, N. 1977. *Applied mathematical demography*. New York: A Wiley-Interscience
14 publication.

15 Lasser, K.E., D.U. Himmelstein, and S. Woolhandler. 2006. "Access to care, health
16 status, and health disparities in the United States and Canada: Results of a cross-national
17 population-based survey." *American Journal of Public Health* 96(7):1300-1307.

18 Lee, R.D. and L.R. Carter. 1992. "Modeling and forecasting United-States mortality."
19 *Journal of the American Statistical Association* 87(419):659-671.

20 Nau, C. and G. Firebaugh. 2012. "A new method for determining why length of life is
21 more unequal in some populations than in others." *Demography*:1-24.

1 Olshansky, S.J., T. Antonucci, L. Berkman, R.H. Binstock, A. Boersch-Supan, J.T.
2 Cacioppo, B.A. Carnes, L.L. Carstensen, L.P. Fried, D.P. Goldman, J. Jackson, M. Kohli,
3 J. Rother, Y. Zheng, and J. Rowe. 2012. "Differences in life expectancy due to race and
4 educational differences are widening, and many may not catch up." *Health affairs*
5 *(Project Hope)* 31(8):1803-1813.

6 Peltzman, S. 2009. "Mortality Inequality." *Journal of Economic Perspectives* 23(4):175-
7 190.

8 Pollard, J.H. 1991. "Fun with Gompertz." *Genus* 47(1/2):1-20.

9 R Development Core Team. 2012. "R: A language and environment for statistical
10 computing." Vienna, Austria: R Foundation for Statistical Computing.

11 Schindler, S., S. Tuljapurkar, J.-M. Gaillard, and T. Coulson. 2012. "Linking the
12 population growth rate and the age-at-death distribution." *Theoretical Population Biology*
13 82(4):244-252.

14 Shkolnikov, V.M., E.M. Andreev, Z. Zhang, J. Oeppen, and J.W. Vaupel. 2011. "Losses
15 of expected lifetime in the United States and other developed countries: Methods and
16 empirical analyses." *Demography* 48(1):211-239.

17 Smits, J. and C. Monden. 2009. "Length of life inequality around the globe." *Social*
18 *Science & Medicine* 68(6):1114-1123.

19 Tuljapurkar, S. 2008. "Mortality declines, longevity risk and aging." *Asia-Pacific Journal*
20 *of Risk and Insurance* 3(1):37-51.

21 Tuljapurkar, S. and R.D. Edwards. 2011. "Variance in death and its implications for
22 modeling and forecasting mortality." *Demographic Research* 24:497-525.

1 Tuljapurkar, S., N. Li, and C. Boe. 2000. "A universal pattern of mortality decline in the
2 G7 countries." *Nature* 405(6788):789-792.

3 UNAIDS. 2013. <http://www.unaids.org/en/dataanalysis/datatools/aidsinfo/>.

4 van Raalte, A.A. and H. Caswell. 2012. "Perturbation analysis of indices of lifespan
5 variability." *MPIDR WORKING PAPER WP 2012-004*.

6 van Raalte, A.A., A.E. Kunst, P. Deboosere, M. Leinsalu, O. Lundberg, P. Martikainen,
7 B.H. Strand, B. Artnik, B. Wojtyniak, and J.P. Mackenbach. 2011. "More variation in
8 lifespan in lower educated groups: evidence from 10 European countries." *International*
9 *Journal of Epidemiology* 40(6):1703-1714.

10 van Raalte, A.A., A.E. Kunst, O. Lundberg, M. Leinsalu, P. Martikainen, B. Artnik, P.
11 Deboosere, I. Stirbu, B. Wojtyniak, and J.P. Mackenbach. 2012. "The contribution of
12 educational inequalities to lifespan variation." *Population Health Metrics* 10.

13 Vaupel, J.W. 1986. "How change in age-specific mortality affects life expectancy."
14 *Population Studies* 40(1):147-157.

15 Vaupel, J.W. and V. Canudas-Romo. 2003. "Decomposing change in life expectancy: A
16 bouquet of formulas in honor of Nathan Keyfitz's 90th birthday." *Demography*
17 40(2):201-216.

18 Vaupel, J.W., Z. Zhang, and A.A. van Raalte. 2011. "Life expectancy and disparity: an
19 international comparison of life table data." *BMJ open* 1(1):e000128.

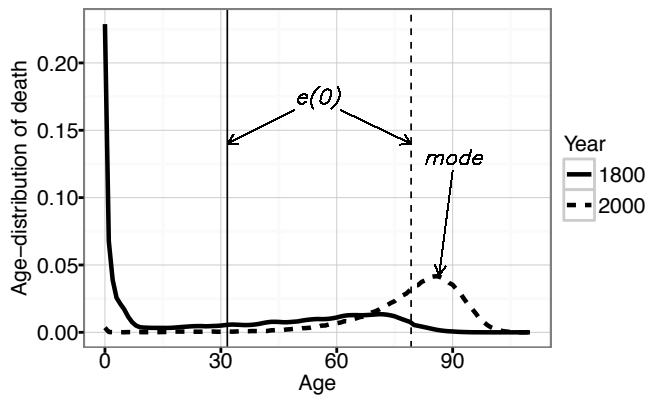
20 Wilmoth, J.R. and S. Horiuchi. 1999. "Rectangularization revisited: Variability of age at
21 death within human populations." *Demography* 36(4):475-495.

1 Wilper, A.P., S. Woolhandler, K.E. Lasser, D. McCormick, D.H. Bor, and D.U.
 2 Himmelstein. 2009. "Health insurance and mortality in US adults." *American Journal of*
 3 *Public Health* 99(12):2289-2295.
 4 Zhang, Z.and J.W. Vaupel. 2009. "The age separating early deaths from late deaths."
 5 *Demographic Research* 20:721-729.

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7 **Figures**

8 **Fig. 1** The period age-distributions of death for years 1800 and 2000 in Sweden. The
 9 vertical lines show life expectancy $e(0)$. The modal age at death in adulthood became
 10 prominent only after the major process of mortality decline

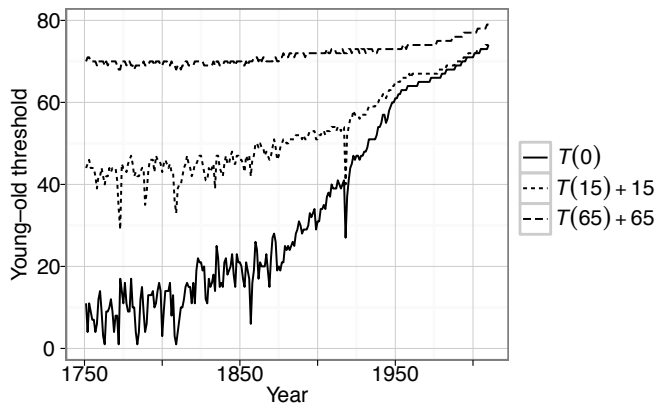


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1 **Fig. 2** The age-specific sensitivities to proportional mortality reduction in Sweden for
 2 1950 (solid lines) and 2011 (dashed lines) of: (a) the variance of age at death after 15 y;
 3 and (b) the mean age at death, i.e. life expectancy, after this age. The vertical lines show
 4 the life expectancy after 15 y, $e(15)+15$



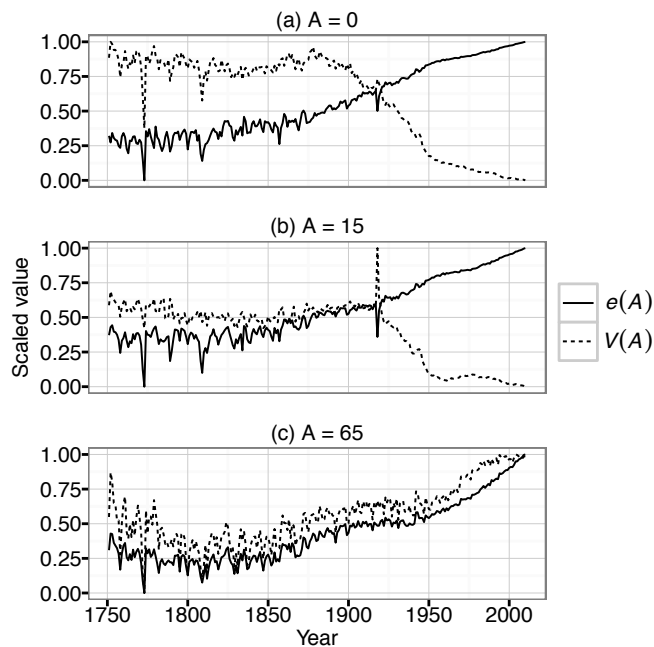
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 8 **Fig. 3** The young-old thresholds, $T(A)+A$ for $A=0, 15$ and 65 y from 1751 to 2011 in
 9 Sweden. Note that even in 2011, $T(65)$ is only 14 years, much smaller than $T(0)$ and
 10 $T(15)$



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2 **Fig. 4** The trajectories of the variance $V(A)$ for $A=0, 15$ and 65 y, in panels (a), (b) and (c)
3 respectively, alongside the corresponding trajectories of life expectancy, $e(A)$. Each
4 trajectory has been scaled so that it ranges between zero and one during the study period



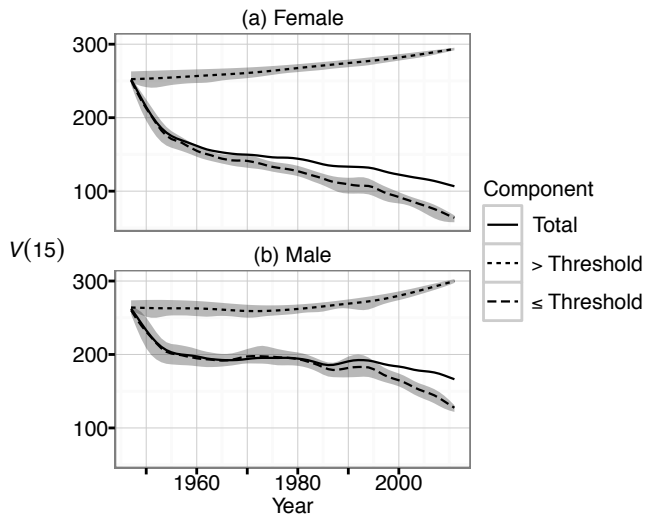
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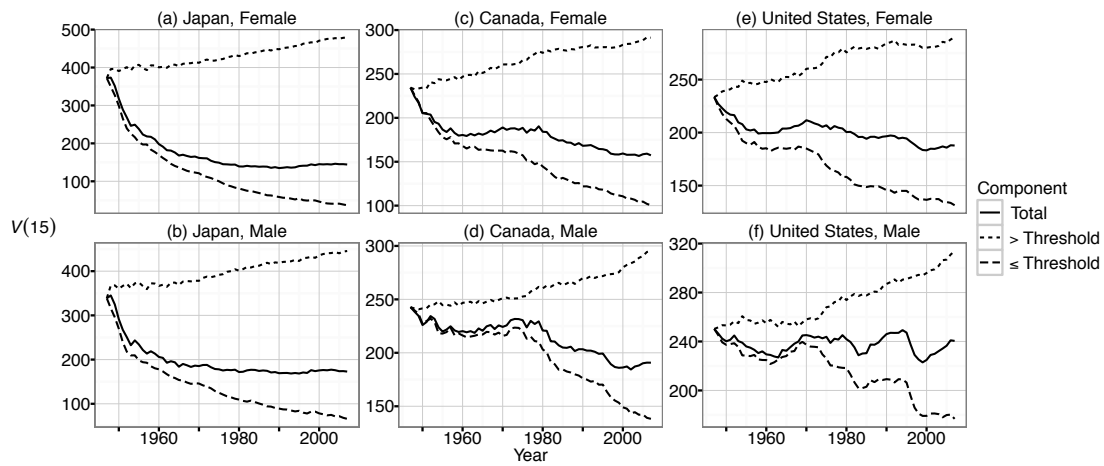
8 **Fig. 5** For females (a) and males (b), solid lines show the average trajectory of the
9 variance of age at death after 15 y, $V(15)$, across 40 regions in the Human Mortality
10 Database (see Table S1 for details). The upper dotted lines show this average trajectory as
11 it would have been with mortality change only above the threshold age, $T(15)+15$. The
12 lower dashed lines show this for mortality change below the threshold. Note that the lines
13 do not show the variance change at each time t ($1947 \leq t \leq 2011$), but show the
14 cumulative effects of mortality change from 1947 to time t . Thus, if W is the annual
15 change in V due to mortality change below or above the threshold in each year, the dotted

1 and dashed lines are given by $V(15, 1947) + \int_{1947}^t W(15, u) du$. The grey shading around the
 2 dotted and dashed lines shows the 95% range among regions of the annual components of
 3 change



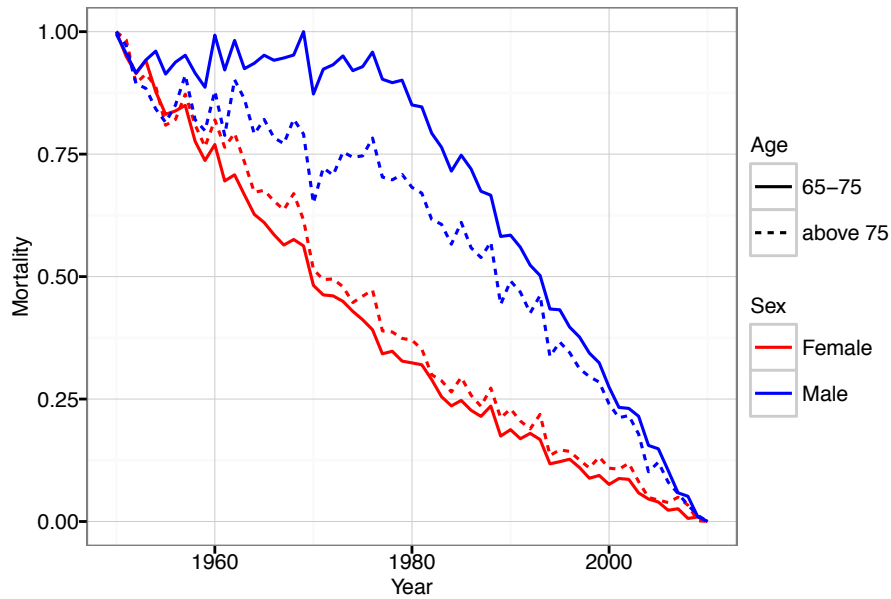
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1 **Fig. 6** Decomposition of the variance of age at death after 15 y, $V(15)$ into the effects of
 2 mortality change below and above the young–old threshold age, $T(15)+15$. Solid lines
 3 show the overall variance change, dashed lines the change produced from ages below the
 4 threshold, and dotted lines the change produced from ages above the threshold. We show
 5 separate decompositions for females and males in Japan (a & b), Canada (c & d) and the
 6 United States (e & f) from 1947 to 2007. Lines show the cumulative variance effects of
 7 mortality change plus $V(15, 1947)$, see details in the legend of Fig. 5



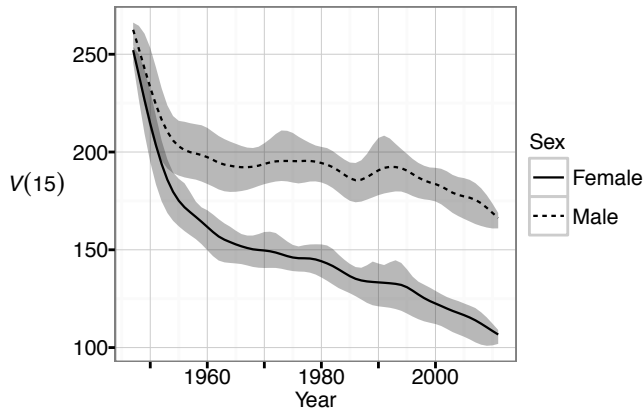
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1 **Fig. S1** Weighted average one-year central death rates between 65–75 years and above 75
2 years for each sex in Sweden from 1950 to 2010. The weights used in the calculation
3 were proportional to the probability of survival to each age. The trajectories have been
4 standardized so that each ranges between zero and one



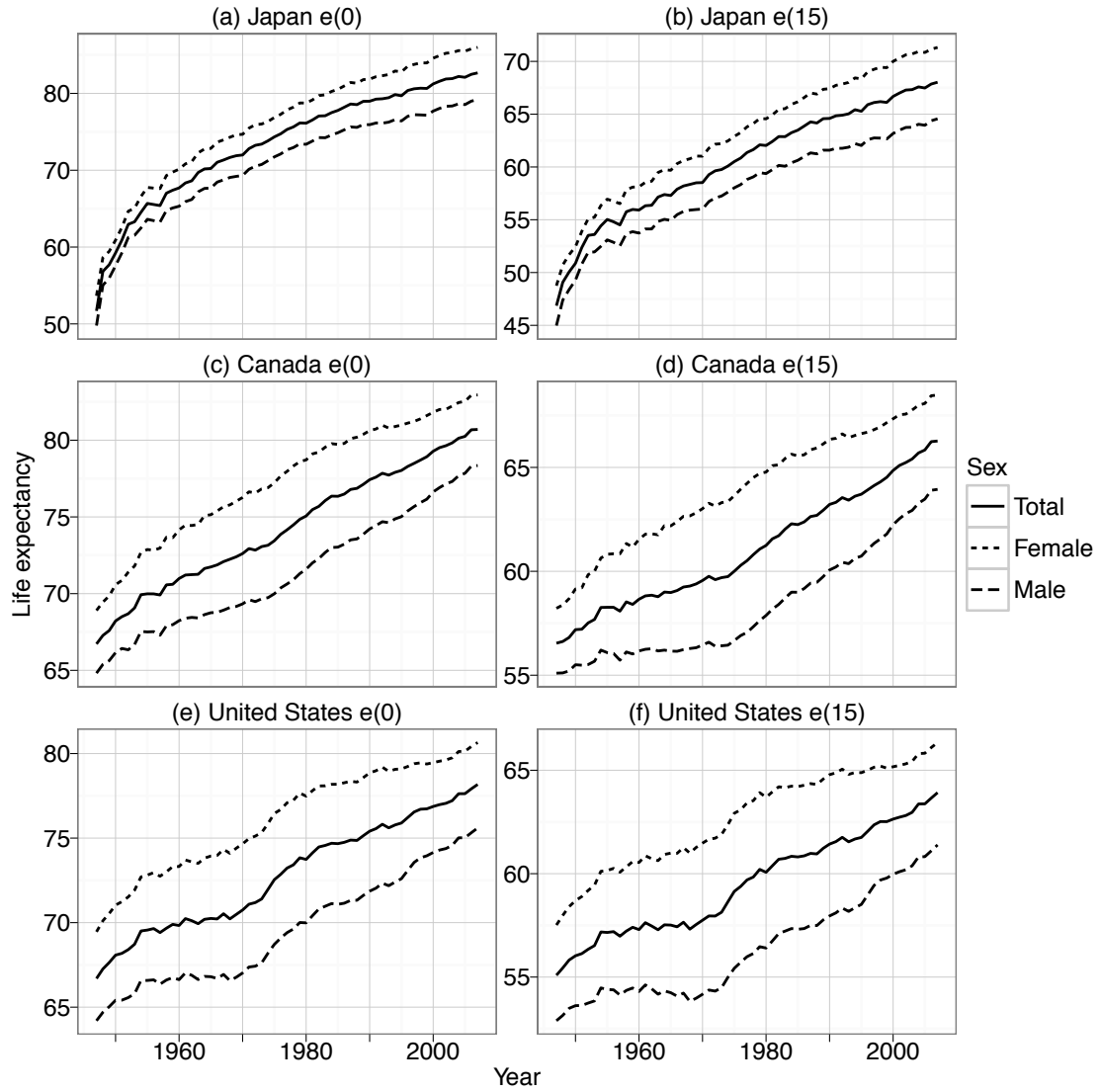
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1 **Fig. S2** The average trajectories of the variance of age at death after 15 years, $V(15)$, in
2 40 regions from the Human Mortality Database for females (solid lines) and males
3 (dashed lines). The grey shading shows the 95% range among regions of the direction of
4 variance change in each year. The maximum extent of years is 1947–2011, but not all
5 regions have data for each year; see details in Table S1



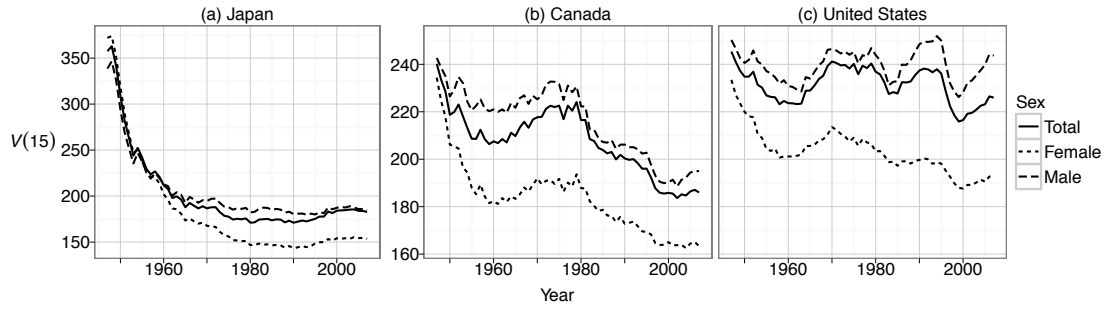
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- 1 **Fig. S3** Life expectancy after zero years i.e. at birth, $e(0)$ and 15 years, $e(15)$ respectively
- 2 in (a) & (b) Japan, (c) & (d) Canada and (e) & (f) the United States from 1947 to 2007.
- 3 Separate lines show the total population, females only and males only



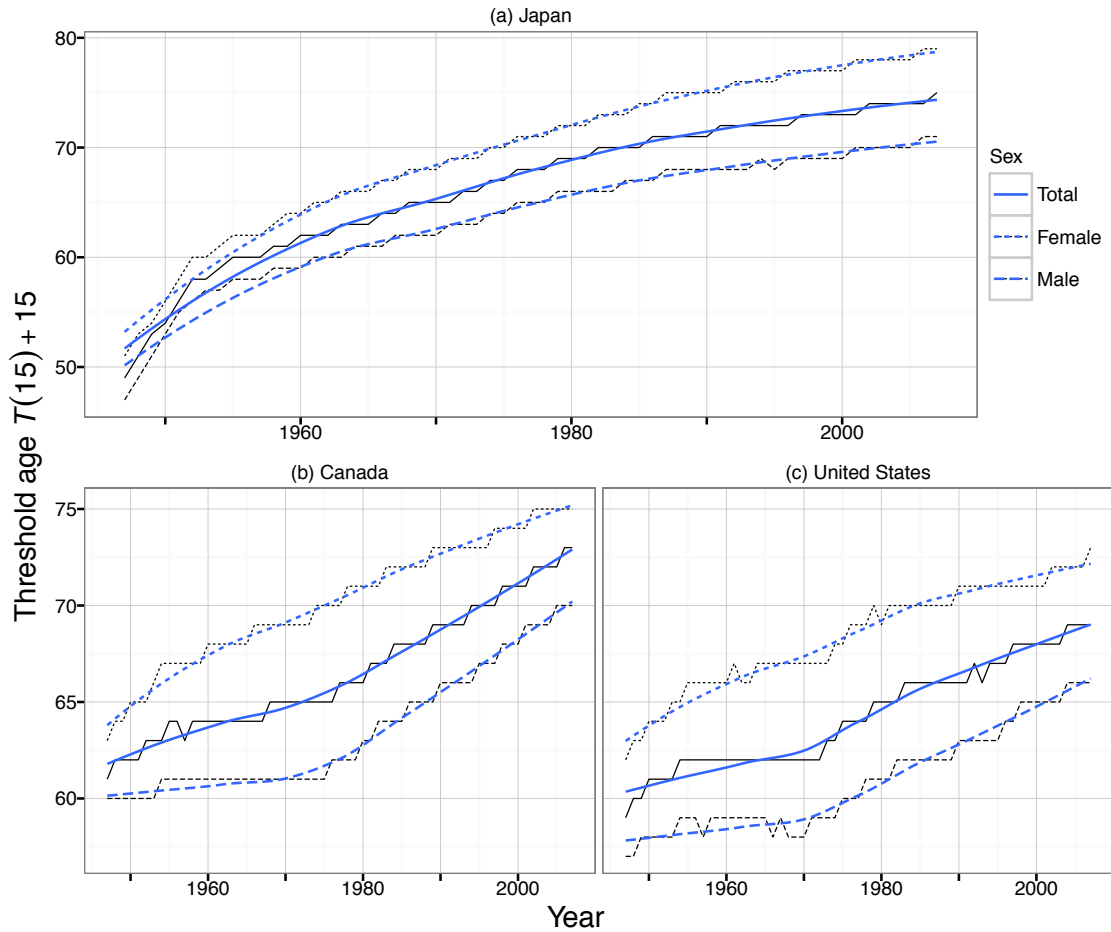
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1 **Fig. S4** The variance of age at death after 15 years, $V(15)$ in (a) Japan, (b) Canada and (c)
2 the United States from 1947 to 2007. Separate lines show the total population, females
3 only and males only



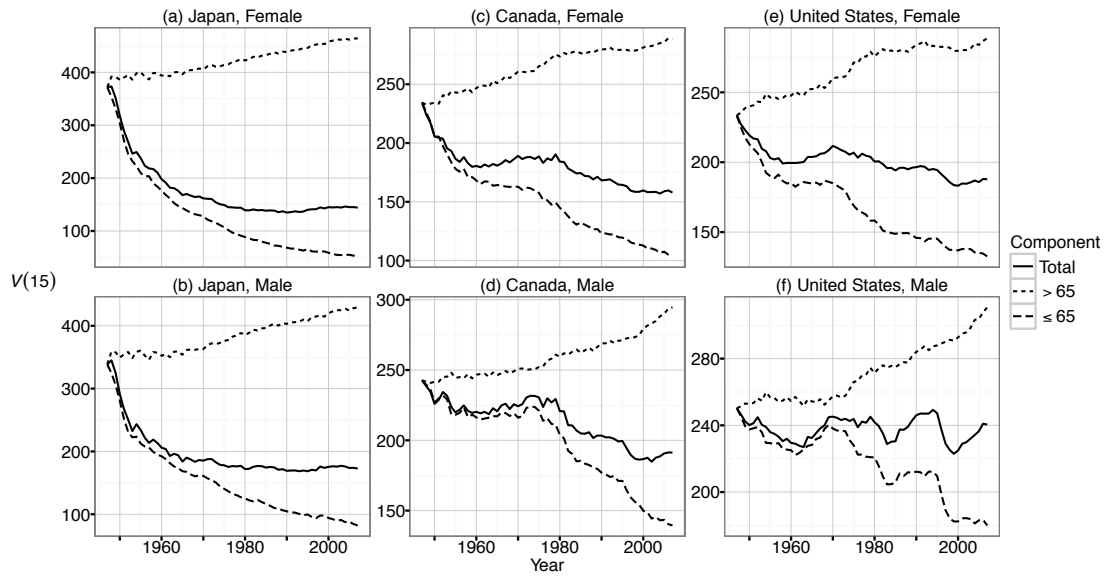
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1 **Fig. S5** The young–old threshold for individuals alive at 15 years, $T(15)+15$, for (a)
 2 Japan, (b) Canada and (c) the United States from 1947 to 2007. Separate lines show the
 3 total population, females only and males only. The blue lines show the fit of a loess
 4 smooth to the integer values that arise from our use of mortality data in discrete one-year
 5 age and period form



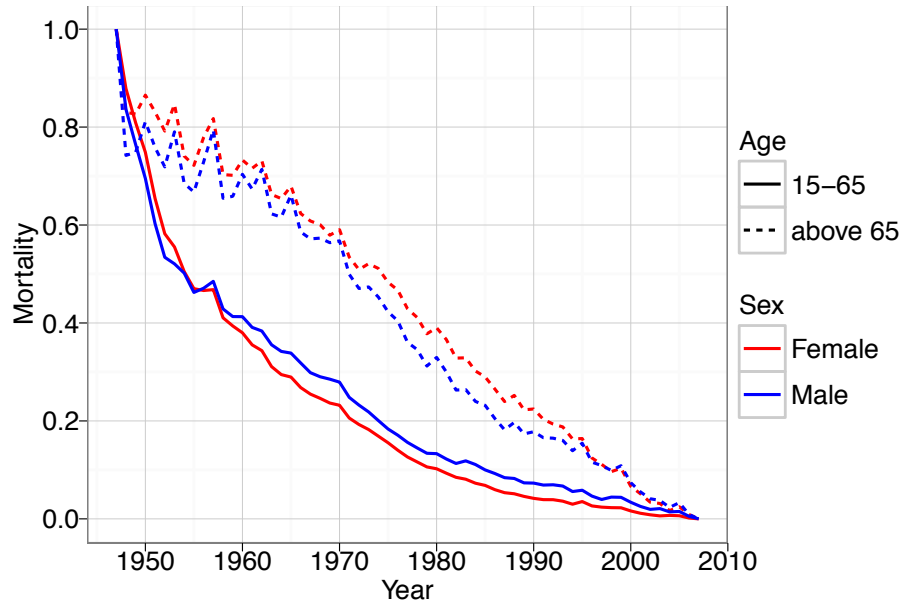
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1 **Fig. S6** Decomposition of the variance of age at death after 15 years, $V(15)$ into the
 2 effects of mortality change at working ages (15–65 years) and retired ages (>65 years).
 3 Solid lines show the total cumulative change, dashed lines the cumulative change
 4 produced by working ages only, and dotted lines the cumulative change produced by
 5 retired ages only. We computed separate decompositions for females and males in Japan
 6 (a & b), Canada (c & d) and the United States (e & f) from 1947 to 2007



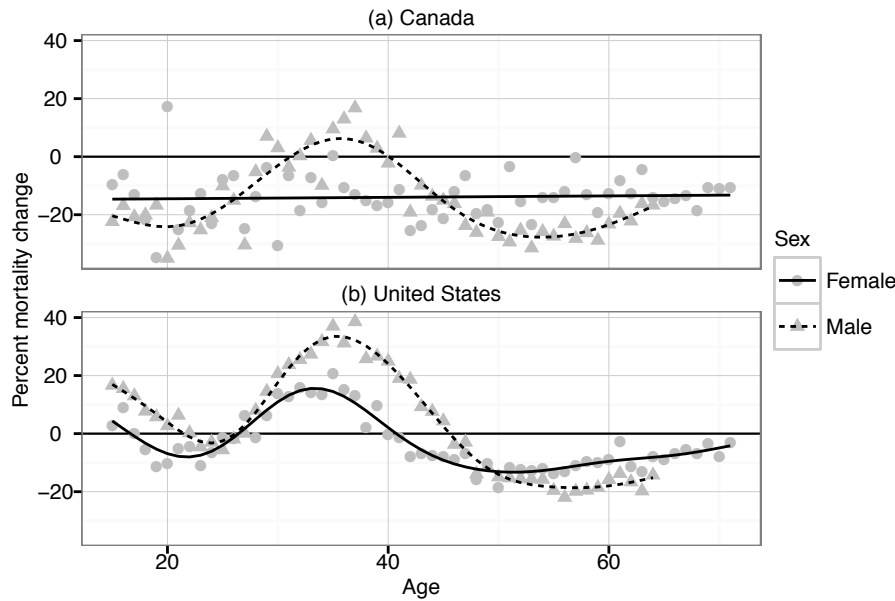
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1 **Fig. S7** Weighted average one-year central death rates between 15–65 years and above 65
2 years for each sex in Japan from 1947 to 2007. The weights used in the calculation were
3 proportional to the probability of survival to each age. The trajectories have been
4 standardized so that each ranges between zero and one



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1 **Fig. S8** The percentage changes to the central one-year death rates at ages from 15 years
 2 to the average young–old threshold for females (71 years) and males (64 years) between
 3 1983 and 1994 in (a) Canada and (b) the United States. Separate lines show the change in
 4 females and males



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Table S1 Regional data series in the Human Mortality Database used in our international comparisons. The earliest and latest years show the data availability for each series in the period 1947–2011

Country or region	Earliest year	Latest year
Australia	1947	2009
Austria	1947	2010
Belgium	1947	2009
Bulgaria	1970	2010
Belarus	1970	2010

Canada	1947	2007
Switzerland	1947	2011
Chile	1992	2005
Czech Republic	1950	2011
West Germany	1956	2010
East Germany	1956	2010
Denmark	1947	2011
Spain	1947	2009
Estonia	1959	2010
Finland	1947	2009
France	1947	2010
England & Wales	1947	2009
Northern Ireland	1947	2009
Scotland	1947	2009
Hungary	1950	2009
Ireland	1950	2009
Iceland	1947	2010
Israel	1983	2009
Italy	1947	2009
Japan	1947	2009
Latvia	1970	2010
Luxembourg	1960	2009
Lithuania	1959	2010
Netherlands	1947	2009
Norway	1947	2009
New Zealand non-Maori	1947	2008
Poland	1958	2009
Portugal	1947	2009

Russia	1959	2010
Slovakia	1950	2009
Slovenia	1983	2009
Sweden	1947	2011
Taiwan	1970	2010
Ukraine	1970	2009
USA	1947	2010

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