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On the Quantum of Fertility: A Bias Correction Approach Using the Slope Information

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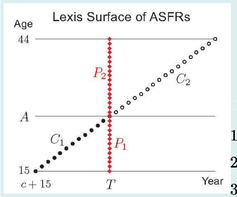
To predict cohort fertility…



$$C = C_1 + C_2 = \sum_{a=15}^{A} f(a, c+a) + \sum_{a=A+1}^{44} f(a, c+a)$$

Some Plain Prediction Methods:

1. Freeze Rates $\quad \widehat{C} = C_1 + P_2$
2. Equal Ratio $\quad \widehat{C} = C_1 + (C_1/P_1)P_2$
3. Freeze Adjusted Rates $\quad \widehat{C} = C_1 + [1 - r(T)]^{-1} P_2$

What we know so far…

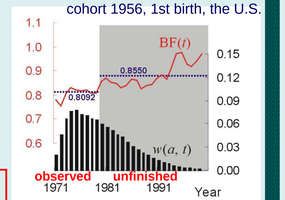
A satisfactory estimate of cohort fertility depends crucially on an accurate prediction of the future trend of period quantum!

Prediction in the many-to-one framework:

$$C_2 = \sum_{a=A+1}^{44} \mathrm{BF}(c+a) w(a, c+a) = \widetilde{\mathrm{BF}} \sum_{a=A+1}^{44} w(a, c+a) = \widetilde{\mathrm{BF}} \sum_{a=A+1}^{44} p(a, T)$$

4. Freeze BF-A $\quad \widehat{C} = C_1 + \overline{\mathrm{BF}} \sum_{a=A+1}^{44} p(a, T)$

cohort 1956, 1st birth, the U.S.



$$p(a,t) = f(a,t)[\mathrm{TFR}(t)]^{-1}$$
$$w(a,t) = [1 - r(t)]p(a,t)$$

Data and Experimental Design

The data employed in this study are ASFRs by one-year period and by single-year age group, taken from the **Human Fertility Database** and the **Eurostat Database** (last updated in March, 2012):

- **907** and **326** completed cohorts for non-parity and parity specific data from **27** countries/areas, including Canada, the U.S., and 23 European countries.
- Each cohort is truncated at ages **16-43** to derive 28 predictions.
- To compare across countries and birth orders, **completed proportion** rather than truncation age is used in analysis.

Prediction error

$$\mathrm{PE} = \frac{\text{est. CTR} - \textbf{true CFR}}{\textbf{true CFR} - \text{obs. CFR}} * 100\%$$

how much of the **unfinished** fertility has not been **correctly** estimated

Mean Absolute Prediction Error

Birth Order	Samples	Conversion Error†	Freeze Rates	Equal Ratio	Freeze BF-A	Freeze BF-L
completed proportion ∈ [10%, 30%)						
1	677	2.34	9.47	7.40	**5.05**	6.39
2	833	2.25	10.92	8.60	**7.43**	9.07
3+	998	2.32	31.29	35.82	34.23	**29.64**
all	2,759	2.48	17.03	14.59	**12.74**	13.02
completed proportion ∈ [30%, 50%)						
1	597	3.36	10.76	7.29	**5.28**	6.90
2	687	2.97	12.52	9.27	**8.02**	10.10
3+	843	2.61	26.17	37.49	32.56	**25.03**
all	2,440	2.98	16.96	14.94	13.38	**13.22**
completed proportion ∈ [50%, 65%)						
1	499	4.42	11.87	7.33	**6.10**	7.84
2	576	3.70	13.77	10.47	**9.30**	11.05
3+	673	3.15	23.42	35.65	34.12	**22.79**
all	1,999	3.65	17.15	15.99	14.69	**13.53**
completed proportion ∈ [65%, 75%)						
1	415	5.10	12.23	7.45	**6.67**	8.35
2	451	4.35	14.35	11.55	**10.61**	10.94
3+	529	3.63	21.19	37.30	35.69	**21.01**
all	1,607	4.41	17.22	16.90	15.85	**13.08**
completed proportion ∈ [75%, 85%)						
1	620	5.58	12.39	7.97	**7.49**	8.84
2	611	5.26	14.73	12.63	11.93	**11.45**
3+	679	4.22	18.66	38.62	37.32	**18.54**
all	2,119	5.42	17.63	18.86	17.98	**13.49**

1-to-1 correspondence

birth order	TFR	BF
all	28.75	20.32
1	24.34	13.07
2	29.43	18.65
3+	29.04	29.88

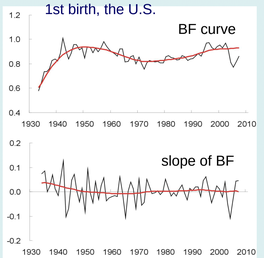
the completed proportion at MAC generally falls between **50%** and **75%**

BF helps to predict cohort fertility, but the quantum effect may cause a BIG bias.

Can Bias be Corrected?

One may extract useful information from the BF curve to effectively correct the prediction bias!

1st birth, the U.S.

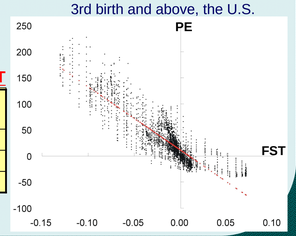


use the **G3GRID** procedure in **SAS code** to create a **smoothed Lexis surface** from the original data

Useful Variables:

- **FST:** the BF slope at the truncation age
  FST2=FST**2
- **SND:** the difference of **FST** at truncation
  SND2=SND**2
- **TAGE:** the truncation age
  TFST, TFST2, TSND, TSND2
- **STRONG:** an indicator if **FST***SND>0
  SFST, SFST2, SSND, SSND2
- **POSITIVE:** an indicator if **FST**>0
  PFST, PFST2, PSND, PSND2

3rd birth and above, the U.S.



Before Correction

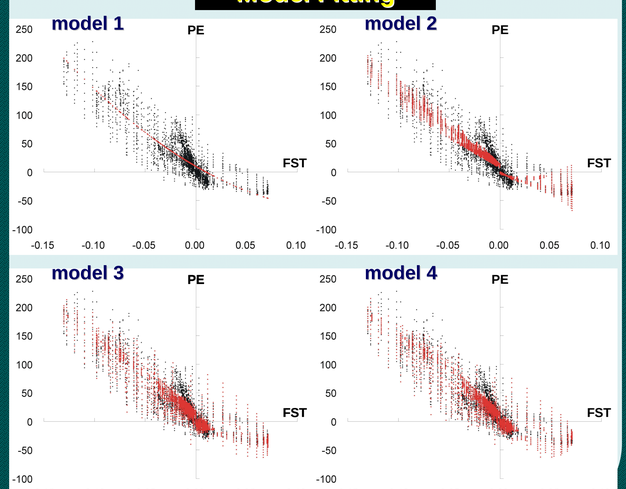
correlation coefficient between PE and FST

Birth Order smoothing	1	2	3+	all
mild	-0.427	-0.577	-0.779	-0.601
medium	-0.497	-0.701	-0.861	-0.743
strong	-0.471	-0.748	**-0.881**	-0.810

Model Setting & Performance

	model 1		model 2		model 3		model 4	
Intercept	V	***	V		V		V	**
FST	V	***	V		V	***	V	***
FST2	V	***	V	***			V	***
SND					V	***	V	***
SND2							V	
TAGE			V	***	V	***	V	***
TFST			V	***	V	***	V	***
TFST2			V	***			V	***
TSND					V	***	V	***
TSND2							V	***
STRONG			V	*	V		V	***
SFST			V	***	V	***	V	***
SFST2			V	***			V	**
SSND					V	***	V	
SSND2							V	***
POSITIVE			V	***	V	***	V	***
PFST			V	***	V	***	V	***
PFST2			V	***			V	***
PSND					V	**	V	***
PSND2							V	
R-square	0.7959		0.8343		0.9080		0.9147	

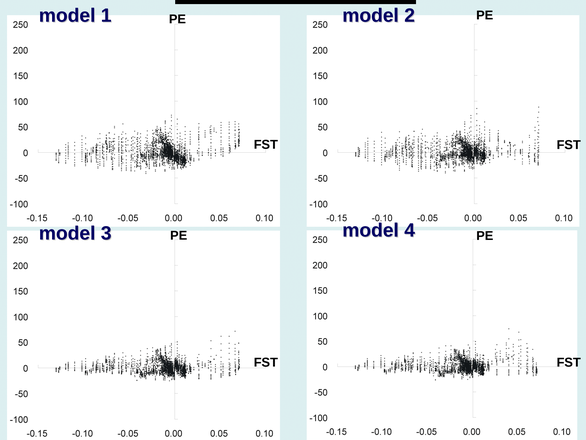
Model Fitting



Correction Technique

$$\widehat{\mathrm{PE}} = \frac{\text{est. CFR} - \textbf{adj. est. CFR}}{\textbf{adj. est. CFR} - \text{obs. CFR}} * 100\%$$

After Correction



Mean Absolute Prediction Error

Birth Order	Samples	Conversion Error†	Freeze Rates	Equal Ratio	Freeze BF-A	Freeze BF-L	adj. Freeze BF-A mild	adj. Freeze BF-A medium	adj. Freeze BF-A strong
completed proportion ∈ [10%, 30%)									
1	677	2.34	9.47	7.40	5.05	6.39	3.81	3.34	3.41
2	833	2.25	10.92	8.60	7.43	9.07	6.09	4.87	3.73
3+	998	2.32	31.29	35.82	34.23	29.64	16.23	11.27	8.83
all	2,759	2.48	17.03	14.59	12.74	13.02	9.52	7.23	4.90
completed proportion ∈ [30%, 50%)									
1	597	3.36	10.76	7.29	5.28	6.90	4.05	3.51	3.32
2	687	2.97	12.52	9.27	8.02	10.10	6.01	4.40	3.15
3+	843	2.61	26.17	37.49	32.56	25.03	13.26	8.69	7.05
all	2,440	2.98	16.96	14.94	13.38	13.22	8.71	6.19	4.15
completed proportion ∈ [50%, 65%)									
1	499	4.42	11.87	7.33	6.10	7.84	4.79	4.47	4.22
2	576	3.70	13.77	10.47	9.30	11.05	6.47	4.55	3.52
3+	673	3.15	23.42	35.65	34.12	22.79	12.48	8.24	6.79
all	1,999	3.65	17.15	15.99	14.69	13.53	8.58	5.76	4.11
completed proportion ∈ [65%, 75%)									
1	415	5.10	12.23	7.45	6.67	8.35	5.59	5.21	4.83
2	451	4.35	14.35	11.55	10.61	10.94	7.07	5.03	4.25
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all	1,607	4.41	17.22	16.90	15.85	13.08	8.98	6.20	4.70
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2	611	5.26	14.73	12.63	11.93	11.45	8.14	5.81	4.90
3+	679	4.22	18.66	38.62	37.32	18.54	14.09	10.43	8.52
all	2,119	5.42	17.63	18.86	17.98	13.49	10.31	7.69	5.64

Bias can be corrected, but note that strong smoothing may encounter an end-point problem, which needs some further refinement.