ECONOMIC INCENTIVES TO REDUCE ABOVE REPLACEMENT FERTILITY: SIMULATING THE QUANTITY-QUALITY TRADEOFF*

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Abstract

The current paper applies the Quantity-Quality model to speed up the demographic transition and investigates incentives for curbing high fertility behavior in developing nations to ease the burden of a rapidly growing population. Earlier empirical studies just explore how greater number of children is usually associated with lower levels of quality but my static model contributes to the existing research by actually testing the Q-Q model for various well-known functional forms, such as the Cobb-Douglas function, the Leontief function and the Stone Geary function which are used frequently in economic modeling. Using simulations with data from India, I estimate the income and price elasticities for the different forms of household utility to demonstrate the change in fertility and schooling. I also run comparative static exercises to analyze the outcome of policy experiments and test the hypothesis that policy initiatives may not always yield anticipated results; one of the key findings is that simply subsidizing qualitative improvements in children (reducing cost of education alone without other investments in family planning etc.) may be insufficient to trigger the Q-Q tradeoff to curtail above replacement TFR. Reduction in parent's out-of-pocket childcare costs to increase quality may prompt greater childbearing as children are now cheaper to raise.

Keywords: above replacement TFR, quantity-quality tradeoff , financial incentive.

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Abstract

The current paper applies the Quantity-Quality model to speed up the demographic transition and investigates incentives for curbing high fertility behavior in developing nations to ease the burden of a rapidly growing population. Earlier empirical studies just explore how greater number of children is usually associated with lower levels of quality but my static model contributes to the existing research by actually testing the Q-Q model for various well-known functional forms, such as the Cobb-Douglas function, the Leontief function and the Stone Geary function which are used frequently in economic modeling. Using simulations with data from India, I estimate the income and price elasticities for the different forms of household utility to demonstrate the change in fertility and schooling. I also run comparative static exercises to analyze the outcome of policy experiments and test the hypothesis that policy initiatives may not always yield anticipated results; one of the key findings is that simply subsidizing qualitative improvements in children (reducing cost of education alone without other investments in family planning etc.) may be insufficient to trigger the Q-Q tradeoff to curtail above replacement TFR. Reduction in parent's out-of-pocket childcare costs to increase quality may prompt greater childbearing as children are now cheaper to raise.

1 Introduction

A rapidly growing population is one of the major challenges faced by several developing countries since it drains their already limited resources. This makes managing the population growth rate vital for national welfare and development. In the words of Garrett Hardin: "The quality of life and the quantity of it are inversely related" and given that mortality rates have already declined, as per the classical demographic transition theory, the only way to curb unsustainable population growth rates, is by reducing fertility. There are several schools of thought on population dynamics ranging from the pro-natalist to passivists and alarmists but this paper does not endorse any particular stand. It just uses the pressures of rapidly expanding population and depletion of nonrenewable resources as a driving impetus for working towards stable growth. Population pressure is a global issue but we must look for solutions locally since each country is unique with respect to its problems and policy needs. Once we identify the background factors that filter into the proximate determinants of fertility, we can propose feasible and effective policy instruments which may potentially incentivize smaller family sizes and reduce the burden of high population. This will help developing countries to speed up the demographic transition and move to a sustainable fertility time-path, which in turn should help reduce the population pressure on our scarce resources. With this ideology in mind, the paper will attempt to investigate some feasible policy instruments to moderate population growth rates in developing countries. Specifically this study contributes to existing research by developing a static model of household behavior and applying various well-known functional forms, such as the Cobb-Douglas, the Leontief and the Stone Geary functions to estimate the Quantity-Quality (Q-Q) model of fertility and compares the impact of different policy experiments.

The paper is organized as follows: Section 2 describes the background and motivation; Section 3 provides an overview of the surveyed literature while Section 4 is devoted to the main research questions; Section 5 defines the methodology for the Static model and reports the analytical results; Section 6 illustrates the simulations exercises along with the data; finally Sections 7, 8, 9 and 10 discuss the policy implications, conclusions, limiting concerns and plans for future extensions. The detailed derivations and graphs for the models are provided in the Appendix.

2 Background & Motivation

One of the fundamental driving forces in the domain of economics is scarcity. Scarcity arises since our infinite demands are constrained by the finiteness of resources available to us. These ever increasing demands are made by the growing population which in turn is a direct result of fertility behavior.

By the standard Demographic Balancing Equation we have:

$$[Population_{t+1} - Population_t](\Delta P) = NaturalIncrease_t(NI) + NetMigration_t(NM)$$

where $NaturalIncrease_t(NI) = Births_t(B) - Deaths_t(D)$

and $NetMigration_t(NM) = Immigration_t(I) - Emigration_t(E)$

Hence Population growth formula is expressed as: $\Delta P \equiv NI + NM = B - D + I - E$

With fertility, mortality and migration being the main components of population growth, Singh et al. (1986) provide an interesting summary of how we make the transition from the problem of high population growth to the proposed solution of incentivizing behavior directed at reduced fertility. Fertility regulation to ensure population growth deceleration can potentially be brought about by improved communication with the general public to influence their demographic predispositions, provision of services to encourage desired behavior, incentives or disincentives to regulate trends and tendencies, appropriate social institutions and opportunities or coercive action by administrative bodies. Given these broad mechanisms, I want to analyze the tradeoff between child quality and quantity to counteract the causes of high population growth and focus on India which is a developing country with above replacement fertility (TFR>2.1).

The emphasis on India is motivated by several factors. With a population of more than 1 billion people, it is the second most populous country in the world. It has a population growth rate of 1.64% and on average, women in India have 2.6 births during their lifetimes. In spite of a booming economy, per capita income is quite low due to the large size of India's population. The current population is relatively young (by 2020, the average age of an Indian is expected to be 29 years), which means that the country may soon face issues where its youthful work force may turn from an asset to a burden as they age and multiply. The common fertility pattern differs from the Ministry of Health and Family Welfare's ideal 2-child family due to early age of marriage, son bias, child labor, lack of formal old age security among other factors. According to the U.S. Census Bureau, based on current trends with the poorest and most populous states consistently growing, India could surpass China and become the world's largest country by 2025.

3 Review of Literature

A lot of research is already underway to better understand the nature of the problem of over-population and excessively high fertility and this is reflected in the academic literature. Some of the recurring themes are:

3.1 Demographic Change and Fertility Transition

Historically most countries in the developed world underwent a massive change in their demographic and social structure which propelled their economies from slow to rapid and sustainable economic growth. There are many studies on the fertility revolution and demand for children like the fundamental work by Becker, Easterlin & Crimmins, Schultz and others; these help to develop the micro foundations for the structural models at the household and institutional level. The goal is to shift fertility control from social and biological motivations to a state where family size is regulated and limited by decisions of the individual households. Conscious fertility control involves averting undesired births so that actual fertility is below the reproductive potential and natural fertility levels but this faces several hurdles like lack of access to family planning services and lack of motivation by parents.

3.2 Quantity-Quality Tradeoff – Theoretical Models & Applications

The seminal article by Becker (1960) determined that demand and supply for children are both generated by parents in the presence of uncertainty which causes divergence between actual and desired fertility. This continued discrepancy in realized and desired births leads to unsustainable population growth rates. Becker & Lewis (1973) investigated the negative correlation between child quality and quantity while Rosenzweig & Wolpin (1980) tested the predictive power of the Q-Q model to show that parents prefer to have similar levels of quality for their children and increasing income reduces quantity and raises quality in the long run. Since shadow prices are unobservable, empirically testing the interaction hypothesis is a complicated task. They use the 'natural experiment' of twin-births since multiple births from a single pregnancy is usually a case of unanticipated rise in child quantity and the substitution effects show that an exogenous increase in family size has a negative effect on expenditure for schooling and other commodities.

Edlefsen's (1983) study looks at the income compensated price effects in the Q-Q models and stresses that if child quantity and quality are assumed to be net substitutes with respect to all other goods and services, then the price effect will always be negative. Second order conditions alone are insufficient to infer whether rising costs make parents substitute away from number of children but the own income compensated price effects are all negative. These are normal goods so demand for fertility decreases with a rise in direct cost of quantity. The impact of a rise in price of the interaction term is difficult to sign since it has both a cross substitution effect and an own substitution effect so raising the cost of the factor that influences both the quantity and the quality of children will not only deter fertility but also reduce investments in child quality.

Millimet & Wang (2011) examined the causal relationship between quantity and quality of children for Indonesia but find little support for the tradeoff. Intra-household resource allocation theory suggests that there is interdependence between choice of number of children and investments in child specific human capital and predicts a negative relationship between the two. The tradition of testing the Q-Q model using twins continues in Li et al. (2008) where after accounting for unobserved family preferences, birth spacing and inter-child reallocation, they show that the Q-Q tradeoff in China is more prominent in rural areas as they have restricted access to public education. Depending on the country and the timeline, different researchers have either verified the Q-Q model or questioned its validity. Most of the empirical evidence that exists in the realm of Q-Q tradeoff in fertility runs unidirectional tests on how rise in family size (unanticipated exogenous shifts like birth of twins) affects parental investments in children's human capital but there is very little work examining the reverse causality. The current research proposes to investigate this very linkage and aims to see how improving child quality in terms of health and education may reduce the demand for child quantity.

3.3 Human Capital Investment, Income and Education

Becker & Tomes' (1986) human capital investment model permits assets, earnings and consumption to be transferred to descendants and after comparing different countries empirically they find that all earnings advantages or disadvantages pewter out within three generations. Becker, Murphy & Tamura (1990) uses endogenous fertility and societies where human capital brings in more proceeds tend to have an abundance of investments and small families; conversely places with limited human capital have larger families with lower investments in each member. Saving across generations occurs either in the form of multiple children, greater investment per child or physical capital accumulation. There are two stable steady state that emerge; one is a Malthusian equilibrium with large families and low human capital while the other is Development equilibrium of small families with growing human and physical capital.

Schulz (2005) studies the link between fertility and income and finds an inverse association between income per adult and fertility, among countries and across households. Fertility is found to be lower among women with higher education or ownership of assets or land. For China, Rosenzweig & Zhang (2009) examine how their population control policy affected investment in children and claim that the One Child Policy only had a modest contribution towards the nation's human capital development. Looking at twinning by birth order, the tradeoff between family size and average child quality net of the endowment deficits shows a negative impact on schooling progress and grades, expectations about college, health indexes due to the extra child; this is a result of the close spacing of twin births which puts pressure on the family budget constraint and inhibits spending on children.

Studying the fertility decline and policy measures in developing countries, Cutright (1983) draws attention to an interesting correlation that we have few examples of nations with low literacy and low fertility, while there are many examples of nations with low literacy and high fertility. The author stresses on the importance of improving health and education in order to make family planning programs successful. Kang (2011) looks at South Korean data to find that Q-Q tradeoffs in educational investments is not gender neutral as girls' education suffers from large family sizes but boys have no adverse impact, this may reflect the son preference that is predominant in Korean households. Intra-household time allocation in education for Philippines is related to birth order as per Ejrnaes & Portner (2004). Philippines implements mandatory primary schooling between ages of 7 and 13 with most elementary schools being public and tuition free and children born later get more benefits than their lower birth order siblings, birth order dominates if families hold land but effects are lower if parents are more educated.

3.4 Programs with Direct impact on Fertility (Instruments, Interventions & Incentives)

There are some direct impact programs that are already in place around the world and these may be replicated or extended for other countries. Tan et al (1978) looks at the fertility reduction program implemented by Singapore in 1973 and investigates their five social disincentive policies. Singapore is the first nation in the world to actually implement direct policies that curb population growth with the disincentives as follows: higher accouchement fees where delivery charges were increased for increasing birth orders; lower school admission priority for children of fourth and higher birth orders; reduced maternity leave where women had to use own annual leave time or take unpaid leave from the third child onwards; revised taxation policy where relief for fourth and subsequent children was withdrawn; new government housing allocation policy which gave low priority to larger families. Gertler & Molyneaux (1994, 2000) analyze the Indonesian fertility decline in the eighties and its causes. The National Family Planning Coordinating Board was instrumental in promoting two-child families by encouraging women to delay marriage and use contraceptives which in turn was supplemented by better education and information dissemination, economic advances and higher disposable income, better transportation, proper contraceptive subsidies, family planning programs and a synchronous supply and demand system of birth control aids. On the other hand, the One Child Policy in China is an extreme example of a family planning policy which relied more on strict enforcement rather than some form of incentive mechanism.

Many developing countries have devoted a lot of resources towards public programs, integrated incentives and family policy to curtail fertility. Kangas (1970) and Hossain (1989) scrutinize some of these policies on population control and health. For Bangladesh, the direct and cross effects of subsidies for family planning and secondary school have been very useful for reducing fertility and raising education; their calculated elasticities confirm that directing resources towards the poorest household will lead to cost effective means of achieving policy goals. Generally though, financial rewards are provided to potential contraceptive users, family planning service personnel etc. but the incentives are individual centric and not for the group or community as a whole. Instead of simply targeting recipients or providers, some other possibilities consist of annual or deferred rewards for reproductive age married couples if they avoid having offspring, assigning savings account to women who go for three or four years without pregnancies, distributing family planning bonds to couples who agree to limit their family size etc.

3.5 Programs with Indirect effect on Fertility (Conditional Cash Transfer)

Some real world applications of the Q-Q model can be seen with the cross program effects of Bosca Escola and PROGRESA in Brazil and Mexico respectively. Denes (2003) looks at the impact of the Bosca Escola where even though fertility reduction was not a direct aim, by improving the quality of life for children through higher education and better health care, they may have initiated a Q-Q tradeoff. In many Latin American countries, primary education is not a priority since child labor is prevalent as it generates substantial supplemental income. The Bosca Escola program attempted to stem the school dropout rates by providing financial compensation to households and along with raising school enrollment and educational attainment it helped regulate fertility behavior. The 2002 IFPRI evaluation of the Programa de Educación, Salud y Alimentación reviews the performance of this large scale government anti-poverty endeavor. With deprivation and malnutrition plaguing the nation, PROGRESA was aimed at supporting families with educational, health and nutritional support so they could pull themselves out of poverty. With proper targeting, they managed to reduce poverty in the poorest section of the population, improve health and nutritional status and communities benefited from positive program effects on schooling as well. Both these programs tried to improve education or economic well-being and their attempts to provide better child quality set off a chain reaction series that could ultimately reduce fertility.

3.6 Country specific case for India

With the current population being disproportionately young, India is beginning to feel the effects of the demographic dividend where its youthful work force may turn from an asset to a burden as they age and multiply. The Indian government has long been concerned with its population growth and this has been mirrored in their explicit population policies. The annual population growth rate in the 1940's was low enough to make the administration believe that India would soon follow the trajectory of the developed nations who witnessed industrialization and rising living standards accompanied by a drop in population size. By the 1950's she became one of the first countries to start a national government sponsored family planning program; unfortunately though India's population continued to rise dramatically as per the 1991 census. Part of this failure results from unrealistic targets and the centralization of the family planning programs which fails to incorporate regional differences. Given India's high population density and above replacement fertility rate, Jain & Nag (1986) review the relationship between female education and fertility and try to suggest the most effective strategy to reorient the Indian educational structure in order to affect fertility. Female education monotonically increases age at marriage and contraceptive use, which in turn decreases fertility and so educational policy should be given high priority as it will yield substantial returns in the long run but this investment may not be the most cost effective means of reducing fertility over the short term.

Past literature provides a strong background to examine the issue of rapid population growth and above replacement fertility rates in the context of the Quantity-Quality model.

4 Research Question

My research will contribute to the existing body of work in several important ways. The rationale behind the study is represented in Figure 1[1] where population and development policy is governed by public and private initiatives and eventually filters down to fertility determinants. Some other relationships that may be examined in this context are whether a tax or a subsidy is more effective, is targeting the child or the mother a better option, should we concentrate on only cost of children (price) or demand for children (quantity) or both etc.

The main objective involves identifying some of the feasible and effective policy instruments and target variables that may help to reduce high fertility and unsustainable population growth rates in developing countries. Specifically how can the Quantity-Quality model of fertility be used in collaboration with other financial incentive schemes and family planning programs to incentivize smaller family sizes in developing countries with above replacement fertility rates? In my static framework, I assess how the Quantity-Quality model on fertility reacts to different functional forms of the household utility function. Further I look at income and price elasticities to compare household responsiveness to different interventions and try to measure the impact of various potential policy reforms. I also test the hypothesis that policy initiatives may not always give the anticipated results and use simulations to examine the effect of certain public subsidy schemes related to investments in child health and education.

Social welfare programs including fertility regulation initiatives must satisfy the three basic criteria: maximum benefit for given expenditure which depends on scale of the scheme and interaction across the program; cost effectiveness irrespective of it being in the private or public

[1]There are more linkages possible but for the purposes of this paper, I am focusing only on a subset of them.

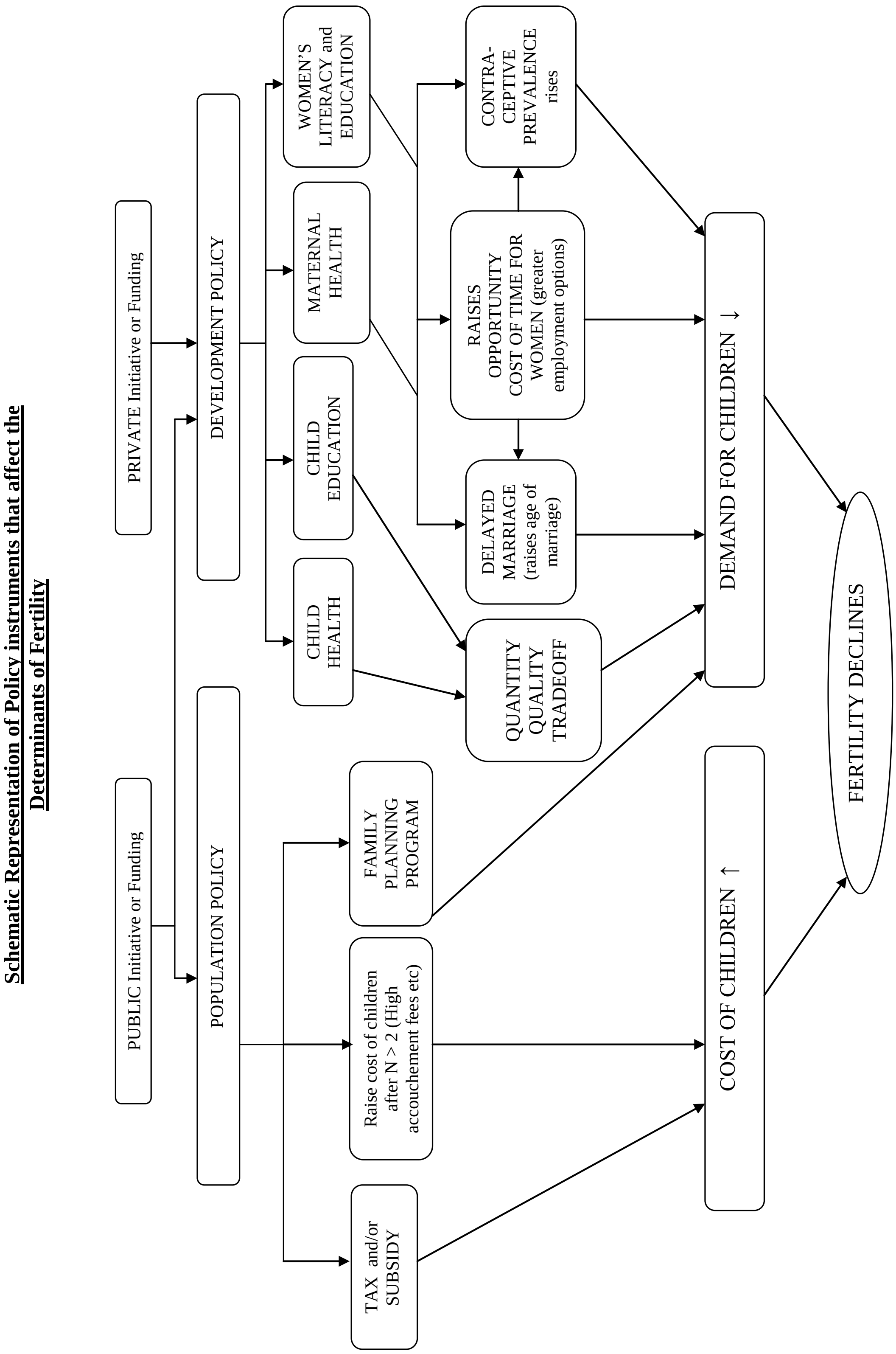


Figure 1: Schematic Representation of Policy Instruments that affect Fertility

sector; and ensuring equity across different disadvantaged groups. Different instruments may act as complements or substitutes and the cross program effects can be tested by comparing the elasticity's of change and counter factual analysis since these proposed schemes may not have any real world counterparts. The social planner or regulatory agency must maximize social welfare but since the parental response is the key variable, we must focus on the decision making individual or the parent to understand policy effectiveness.

5 Methodological Framework

Following the seminal article by Becker (1960), children can be interpreted as durable goods that yields some return, and demand for children is made in the presence of uncertainty regarding the gender of the child, survival probability and success as an adult, which can lead to a divergence between actual and desired fertility. Demand and supply are both generated by the same agents, i.e. parents, and their decision is affected by income, tastes, knowledge and costs. The parental decision-making process with respect to childbearing is an optimization exercise in which parents choose the number of children (quantity) and health and educational status of their children (quality) but in the static analysis, we cannot separate out spacing of births in a household.

5.1 General Theoretical Model

A general framework for the Quantity-Quality model for a parent or household follows:

$$\max_{n,q,y} U = U(n, q, y)$$

subject to

$$I = p_n.n + p_q.q + p_{nq}.nq + p_y.y$$

where (U) is the Utility function, (n) is the quantity of children, (q) is the quality measure where all children are homogenous and (y) is a composite of all other market commodities; (p_n) represents market price of goods and services for each child independent of child quality (food, shelter), (p_q) stands for investment cost in quality of child independent of number of children (computer, car ride to school), (p_{nq}) is the interaction term for cost of price of quality inputs per child so its value rises with both q and n (tuition fees for school) and finally (p_y) is the price of market goods and services; also (I) is full money income while (R) is real income of the household; (π_i) stands for the shadow cost values with $i = n, q, nq$.

The marginal costs of quantity and quality are represented by the respective shadow price values, i.e. $MC_n = \pi_n = p_n + p_{nq}q$ and $MC_q = \pi_q = p_q + p_{nq}n$ while that for all other market goods and services simply is $MC_y = \pi_y = p_y$. Here the budget constraint is nonlinear and can be modified to: $R = \pi_n n + \pi_q q + p_y y = I + p_{nq}nq$.

The solutions to the primal problem:$n = n^*(\pi_n, \pi_q, p_y, R)$; $q = q^*(\pi_n, \pi_q, p_y, R)$; $y = y^*(\pi_n, \pi_q, p_y, R)$ are plugged into the Indirect Utility function:

$$\psi = \psi(n^*(\pi_n, \pi_q, p_y, R), q^*(\pi_n, \pi_q, p_y, R), y^*(\pi_n, \pi_q, p_y, R)) = \psi(\pi_n, \pi_q, p_y, R)$$

which defines the amount of utility we attain after consumption is optimized. Then using Roy's identity we can solve for the Marshallian demands as follows:

$$\frac{-\frac{\partial\psi(\pi_n,\pi_q,p_y,R)}{\partial\pi_n}}{\frac{\partial\psi(\pi_n,\pi_q,p_y,R)}{\partial R}} = n(\pi_n, \pi_q, p_y, R); \frac{-\frac{\partial\psi(\pi_n,\pi_q,p_y,R)}{\partial\pi_q}}{\frac{\partial\psi(\pi_n,\pi_q,p_y,R)}{\partial R}} = q(\pi_n, \pi_q, p_y, R); \frac{-\frac{\partial\psi(\pi_n,\pi_q,p_y,R)}{\partial p_y}}{\frac{\partial\psi(\pi_n,\pi_q,p_y,R)}{\partial R}} = y(\pi_n, \pi_q, p_y, R)$$

Using both nominal and shadow costs of quality and quantity and ignoring gender/birth-order differences as per Becker & Lewis (1973) and Edlefsen (1983), I find that raising child quality is costly because if there are more children, the same investment has to be made in each of them to ensure homogeneous quality and increasing quantity is costly because if each child is of higher quality, this makes the additional unit more expensive. I then try to compare the different utility functions to distinguish the kind of tradeoffs that result from the underlying utility mechanism. Imposing conditions of Homogeneity, Cournot aggregation and Engel aggregation, I solve the system and conduct some comparative static exercises after setting the model parameters. The own price, cross price and income elasticity values can be estimated to provide insights into the households fertility behavior and that could have important policy prescriptions.

5.2 Functional Specification for Static Model

The general Quantity-Quality model can be applied to different utility functions (Cobb Douglas, Leontief and Stone Geary or Linear Expenditure). The decision making parents maximize their objective function and calculate their optimal consumption for each case as per the procedure described in Section 5.1. The functional forms under consideration are:

Cobb Douglas case

Utility: $U = n^{\beta_n} q^{\beta_q} y^{\beta_y}$

Indirect Utility: $\psi = R(\frac{\beta_n}{\pi_n})^{\beta_n}(\frac{\beta_q}{\pi_q})^{\beta_q}(\frac{\beta_y}{P_y})^{\beta_y}$

Demand Equations:

$n = \frac{\beta_n R}{\pi_n}$

$q = \frac{\beta_q R}{\pi_q}$

$y = \frac{\beta_y R}{\pi_y}$

Leontief Utility case

Utility: $U = \min\{\frac{n}{\beta_n}, \frac{q}{\beta_q}, \frac{y}{\beta_y}\}$

Indirect Utility: $\psi = \frac{R}{\beta_n\pi_n+\beta_q\pi_q+\beta_y P_y}$

Demand Equations:

$n = \frac{\beta_n R}{\beta_n\pi_n+\beta_q\pi_q+\beta_y P_y}$

$q = \frac{\beta_q R}{\beta_n\pi_n+\beta_q\pi_q+\beta_y P_y}$

$y = \frac{\beta_y R}{\beta_n\pi_n+\beta_q\pi_q+\beta_y P_y}$

Stone Geary Utility case

Utility: $U = (n-\gamma_n)^{\beta_n}(q-\gamma_q)^{\beta_q}(y-\gamma_y)^{\beta_y}$

Indirect Utility: $\psi = [R - \pi_n\gamma_n - \pi_q\gamma_q - P_y\gamma_y](\frac{\beta_n}{\pi_n})^{\beta_n}(\frac{\beta_q}{\pi_q})^{\beta_q}(\frac{\beta_y}{P_y})^{\beta_y}$

Demand Equations:

$n = \gamma_n + (\frac{\beta_n}{\pi_n})[R - \pi_n\gamma_n - \pi_q\gamma_q - P_y\gamma_y]$

$q = \gamma_q + (\frac{\beta_q}{\pi_q})[R - \pi_n\gamma_n - \pi_q\gamma_q - P_y\gamma_y]$

$y = \gamma_n + (\frac{\beta_y}{P_y})[R - \pi_n\gamma_n - \pi_q\gamma_q - P_y\gamma_y]$

5.3 Static Model Analytical Solutions:

Here I consider the static version of the Quantity-Quality model where we cannot separate out the spacing of births in a household and the analytical solutions show how the relevant demand functions change with respect to the exogenous price and income parameters. Since the financial instrument (tax, subsidy etc.) will be applied to the prices and must impact fertility so the partial derivatives of interest are: $\partial lnn/(\partial lnp_{nq})$ and $\partial lnn/(\partial lnp_n)$; where the former expression captures the impact on fertility from changing the price of goods and services for children that affect both quantity and quality and the latter shows the impact of raising the price of child quantity on the demand for children.

Notation for Budget Shares and Elasticities:-

$\theta_{nq} = \frac{P_{nq}nq}{R}, \theta_n = \frac{n\pi_n}{R}, \theta_q = \frac{q\pi_q}{R}, \theta_y = \frac{yP_y}{R}; \widetilde{S}_n = \frac{P_n n}{R}; \widetilde{S}_q = \frac{P_q q}{R}$

$e_{nn} = \frac{\partial n/n}{\partial \pi_n/\pi_n}, e_{nq} = \frac{\partial n/n}{\partial \pi_q/\pi_q}, e_{ny} = \frac{\partial n/n}{\partial P_y/P_y}, e_{nR} = \frac{\partial n/n}{\partial R/R};$

$e_{qn} = \frac{\partial q/q}{\partial \pi_n/\pi_n}, e_{qq} = \frac{\partial q/q}{\partial \pi_q/\pi_q}, e_{qy} = \frac{\partial q/q}{\partial P_y/P_y}, e_{qR} = \frac{\partial q/q}{\partial R/R};$

$e_{yn} = \frac{\partial y/y}{\partial \pi_n/\pi_n}, e_{yq} = \frac{\partial y/y}{\partial \pi_q/\pi_q}, e_{yy} = \frac{\partial y/y}{\partial P_y/P_y}, e_{yR} = \frac{\partial y/y}{\partial R/R}.$

General Utility function

- $$\frac{\partial lnn}{\partial lnP_n} = \frac{P_n(e_{nn} - \theta_{nq}e_{qr}e_{nn} + \theta_{nq}e_{nr}e_{qn})}{\pi_n\Big[1 - \theta_{nq}e_{qr} - \theta_{nq}e_{nr} - \frac{P_{nq}qe_{qn}}{\pi_n} - \frac{P_{nq}ne_{qn}}{\pi_q} + \frac{\theta_{nq}e_{nR}e_{qn}P_{nq}q}{\pi_n} + \frac{\theta_{nq}e_{qR}e_{nq}P_{nq}n}{\pi_q}}$$

$$- \frac{\theta_{nq}e_{nR}e_{qq}P_{nq}n}{\pi_q} - \frac{\theta_{nq}e_{nn}e_{qR}P_{nq}q}{\pi_n} + \frac{e_{nq}e_{qn}P_{nq}^2 nq}{\pi_n\pi_q} - \frac{e_{nn}e_{qq}P_{nq}^2 nq}{\pi_n\pi_q}\Big]$$

- $$\frac{\partial lnn}{\partial lnP_{nq}} = \frac{\Big[e_{nR}\theta_{nq} + \frac{(1 - e_{qR}\theta_{nq})e_{nn}P_{nq}q}{\pi_n} + \frac{(1 - e_{qR}\theta_{nq})e_{nq}P_{nq}n}{\pi_q} + \frac{e_{nR}\theta_{nq}e_{nq}P_{nq}q}{\pi_n} + \frac{e_{nR}\theta_{nq}e_{qq}P_{nq}n}{\pi_q}\Big]}{\Big[1 - e_{qR}\theta_{nq} - e_{nR}\theta_{nq}\Big]}$$

Cobb Douglas Utility function

- $$\frac{\partial lnn}{\partial lnP_n} = \frac{-P_n(1 - \theta_{nq})}{\pi_n\Big[1 - 2\theta_{nq} + \frac{\theta_{nq}P_{nq}n}{\pi_q} + \frac{\theta_{nq}P_{nq}q}{\pi_n} - \frac{P_{nq}^2 nq}{\pi_n\pi_q}\Big]}$$

- $$\frac{\partial lnn}{\partial lnP_{nq}} = \frac{\Big[\theta_{nq} - \frac{P_{nq}q}{\pi_n} - \frac{\theta_{nq}P_{nq}n}{\pi_q} + \frac{P_{nq}^2 nq}{\pi_n\pi_q}\Big]}{\Big[1 - 2\theta_{nq} + \frac{\theta_{nq}P_{nq}n}{\pi_q} + \frac{\theta_{nq}P_{nq}q}{\pi_n} - \frac{P_{nq}^2 nq}{\pi_n\pi_q}\Big]}$$

Leontief Utility function

- $$\frac{\partial lnn}{\partial lnP_n} = \frac{-\beta_n P_n}{[(1 - \theta_{nq})(\beta_n\pi_n + \beta_q\pi_q + \beta_y P_y) + \beta_n P_{nq}q + \beta_q P_{nq}n]}$$

- $$\frac{\partial lnn}{\partial lnP_{nq}} = \frac{\theta_{nq}(\beta_n\pi_n + \beta_q\pi_q + \beta_y P_y) - \beta_n qP_{nq} - \beta_q nP_{nq}}{[(1 - \theta_{nq})(\beta_n\pi_n + \beta_q\pi_q + \beta_y P_y) + \beta_n P_{nq}q + \beta_q P_{nq}n]}$$

Stone Geary Utility function

- $$\frac{\partial lnn}{\partial lnP_n} = \Big[\beta_n(\frac{P_n}{\pi_n})(\frac{R}{n\pi_n})\{-1 + (\frac{\gamma_q}{q})(\frac{q\pi_q}{R}) + (\frac{\gamma_y}{y})(\frac{yP_y}{R})\}\{1 - \beta_q(\frac{P_{nq}n}{\pi_q})(1 - \frac{\gamma_n}{n})\}$$

$$-\beta_q(\frac{\gamma_n}{n})(\frac{R}{\pi_q q})(\frac{n\pi_n}{R})(\frac{P_n}{\pi_n})\beta_n(\frac{P_{nq}q}{\pi_n})\{1 - \frac{R}{n\pi_n} + (\frac{\gamma_q}{q})(\frac{q\pi_q}{R})(\frac{R}{n\pi_n}) + (\frac{\gamma_y}{y})(\frac{yP_y}{R})(\frac{R}{n\pi_n})\}\Big]$$

$$/\Big[\{1 - \beta_n(\frac{P_{nq}q}{\pi_n})(1 - \frac{\gamma_q}{q})\}\{1 - \beta_q(\frac{P_{nq}n}{\pi_q})(1 - \frac{\gamma_n}{n})\} - \beta_n\beta_q(\frac{P_{nq}n}{\pi_q})(\frac{P_{nq}q}{\pi_n})\{1 - \frac{R}{q\pi_q}$$

$$+(\frac{\gamma_n}{n})(\frac{n\pi_n}{R})(\frac{R}{q\pi_q}) + (\frac{\gamma_y}{y})(\frac{yP_y}{R})(\frac{R}{q\pi_q})\}\{1 - \frac{R}{n\pi_n} + (\frac{\gamma_q}{q})(\frac{q\pi_q}{R})(\frac{R}{n\pi_n}) + (\frac{\gamma_y}{y})(\frac{yP_y}{R})(\frac{R}{n\pi_n})\}\Big]$$

- $$\frac{\partial lnn}{\partial lnP_n} = \Big[\beta_n(\frac{P_{nq}q}{\pi_n})\{1 - \frac{R}{n\pi_n} + (\frac{\gamma_q}{q})(\frac{q\pi_q}{R})(\frac{R}{n\pi_n}) + (\frac{\gamma_y}{y})(\frac{yP_y}{R})(\frac{R}{n\pi_n})\}\{1 - \beta_q(\frac{P_{nq}n}{\pi_q})(1 - \frac{\gamma_n}{n})\} + \beta_n(\frac{P_{nq}n}{\pi_q})(\frac{P_{nq}q}{\pi_n})$$

$$*\beta_q\{1-\frac{R}{q\pi_q}+(\frac{\gamma_n}{n})(\frac{n\pi_n}{R})(\frac{R}{q\pi_q})+(\frac{\gamma_y}{y})(\frac{yP_y}{R})(\frac{R}{q\pi_q})\}\{1-\frac{R}{n\pi_n}+(\frac{\gamma_q}{q})(\frac{q\pi_q}{R})(\frac{R}{n\pi_n})+(\frac{\gamma_y}{y})(\frac{yP_y}{R})(\frac{R}{n\pi_n})\}\Big]$$

$$/\Big[\{1-\beta_n(\frac{P_{nq}q}{\pi_n})(1-\frac{\gamma_q}{q})\}\{1-\beta_q(\frac{P_{nq}n}{\pi_q})(1-\frac{\gamma_n}{n})\}-\beta_n\beta_q(\frac{P_{nq}n}{\pi_q})(\frac{P_{nq}q}{\pi_n})\{1-\frac{R}{q\pi_q}$$

$$+(\frac{\gamma_n}{n})(\frac{n\pi_n}{R})(\frac{R}{q\pi_q})+(\frac{\gamma_y}{y})(\frac{yP_y}{R})(\frac{R}{q\pi_q})\}\{1-\frac{R}{n\pi_n}+(\frac{\gamma_q}{q})(\frac{q\pi_q}{R})(\frac{R}{n\pi_n})+(\frac{\gamma_y}{y})(\frac{yP_y}{R})(\frac{R}{n\pi_n})\}\Big]$$

The analytical solutions depict how changing the price of quantity or quality affects fertility rates. However there could be several stages in the underlying mechanism that are not evident from the elasticity expression alone; making it impossible to sign the partial derivatives of the demand functions with respect to exogenous parameters. The implication of these partials cannot be determined in the general case, for instance raising the cost of education for children (P_{nq}) could either make parents reduce investment in quality as schooling becomes prohibitively costly and raise quantity or conversely parents may react to costlier education by reducing childbearing hence the net result depends on the strength of the individual effects. So I use the three basic functional forms, assign parameter values and then solve for the magnitude and direction of change to see how the number of children and quality of their education is affected as prices fluctuate.

6 Empirical Estimation via Simulation

Since the impact of variation in prices and income cannot be ascertained directly from the analytical solutions due to the substitutability between child quality and quantity, the estimation strategy used for empirical analysis involves running simulations for each system generated by the different forms of the utility function using national level data from India.

6.1 Data

The numerical analysis and resulting arguments in this study are applicable for any developing country with above replacement fertility rates but the models are specifically tested for India. The study region has 35 main administrative divisions (28 States and 7 Union Territories) and reflects a great deal of heterogeneity with respect to its historical background, geographical features, demographic factors, cultural norms and economic practices; this diversity is reflected in the large scale spatial variation with respect to the fertility transition. The variables employed in this study are mainly demographic variables like fertility; adult and child consumption; educational parameters and human capital investment; along with employment hours and wage rates; income measures and market interest rates for discounting. The primary data source is the National Sample Survey Organization's 64th Round of Household Consumer Expenditure in India (2007-2008) and the World Bank database on national level development indicators for India (1961-2010); supporting material is also collected from Statistical Reports and Bulletins from the Government of India (GOI), Planning Commission; Census of India (2001) from the GOI, Ministry of Home Affairs.

6.2 Simulation Exercise

The three static model systems[2] with different functional forms of utility are solved by running multiple iterations after selecting the exogenous parameters. Given the form of the utility function, the corresponding demand equations, marginal cost relations and budget constraint can be used to construct a system of 6 equations in 6 unknowns from which we can arrive at solutions for the partial derivatives. Each simulation exercise solves for the set of endogenous variables $(n, q, y, \pi_n, \pi_q, R)$ given prices (P_n, P_q, P_{nq}, P_y), income (I) and shares of each item $(\beta_n, \beta_q, \beta_y)$. In addition to these parameters, the Stone Geary form has a linear expenditure function where each good has an associated subsistence requirement $(\gamma_n, \gamma_q, \gamma_y)$. Derivation of the discounted present values of the variables is briefly described below.

The time line for an individual is constructed for a less developed country keeping in mind the lower life expectancy, fewer years of compulsory education etc. The lifecycle design can be expressed by separating out the different stages of life as per Figure 2 where:

Times: $T_0 = 0 \rightarrow$ Born; $T_1 = 18 \rightarrow$ Become young adult and start work; $T_2 = 20 \rightarrow$ Have children; $T_3 = 26 \rightarrow$ Children start school; $T_4 = 38 \rightarrow$ Children finish school; $T_5 = 60 \rightarrow$ Become old adult and retire from work; $T_6 = 65 \rightarrow$ Die.

Periods: $T_0 - T_1 \rightarrow$ Childhood; $T_1 - T_5 \rightarrow$ Young adulthood; $T_5 - T_6 \rightarrow$ Old adulthood.

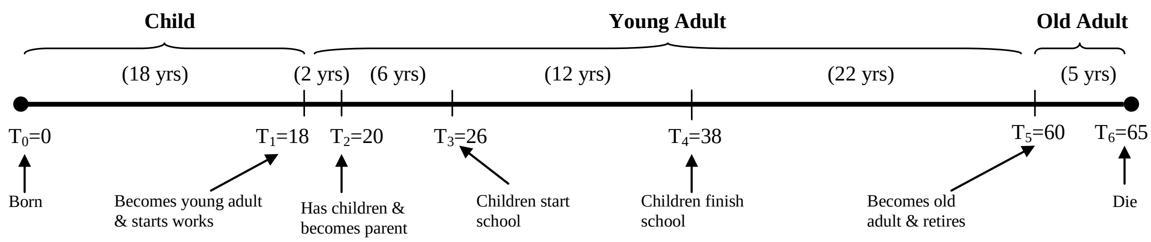


Figure 2: Lifecycle of an individual in a household

For the common parameters, after normalizing the cost of market consumption basket for each adult (P_y), I selected the system parameters using household expenditure data from 2007-08. All cumulative values of prices and quantities are discounted using the market real interest rate of 6.87% for the year 2007 as per the time line described above, which acts as a proxy for social rate of time preference. The lifetime earnings (I) for an adult parent is calculated at the present value for 2007; cost of schooling per year (P_{nq}) refers to the 2006-07 mean educational expenses for rural and urban areas on tuition, exam fees, uniform and coaching; cost of food, clothing and shelter for entire childhood (P_n) is estimated using the fraction of consumption allocated for children in a household; cost of educational expenses excluding school fees per year (P_q) is set at the rural-urban average for per capita non-school expenditure for 2006-07 including books, stationery, transport and other expenses.

[2]System of equations and unknowns for each functional form of utility is described in Appendix I.

Now the budget shares for the three goods ($S_n = \frac{\pi_n n}{R} = 0.015896903, S_q = \frac{\pi_q q}{R} = 0.002507036, S_y = \frac{P_y y}{R} = 0.981596102$) add to unity ($\sum S_i = 1$). For the Cobb Douglas Specific Parameters, the beta values are equal to the budget shares for the three goods ($\beta_n = S_n, \beta_q = S_q, \beta_y = S_y$) and add to unity with ($\beta_n + \beta_q + \beta_y = 1$). In the case of the Leontief Specific Parameters, the budget shares ($S_n = \frac{\beta_n \pi_n}{\beta_n \pi_n + \beta_q \pi_q + \beta_y P_y}, S_q = \frac{\beta_q \pi_q}{\beta_n \pi_n + \beta_q \pi_q + \beta_y P_y}, S_y = \frac{\beta_y P_y}{\beta_n \pi_n + \beta_q \pi_q + \beta_y P_y}$) can be used to residually calculate the beta values ($\beta_n, \beta_q, \beta_y$) after normalizing the share for all non-child goods and services with ($\frac{\beta_n}{n} = \frac{\beta_q}{q} = \frac{\beta_y}{y}$). With the Stone Geary Specific Parameters, the budget shares for each good ($S_n = \frac{\pi_n \gamma_n}{R} + \beta_n \frac{[R - \pi_n \gamma_n - \pi_q \gamma_q - P_y \gamma_y]}{R}, S_q = \frac{\pi_n \gamma_n}{R} + \beta_n \frac{[R - \pi_n \gamma_n - \pi_q \gamma_q - P_y \gamma_y]}{R}, S_y = \frac{\pi_n \gamma_n}{R} + \beta_n \frac{[R - \pi_n \gamma_n - \pi_q \gamma_q - P_y \gamma_y]}{R}$) are solved for the beta values ($\beta_n, \beta_q, \beta_y$) with ($\beta_n + \beta_q + \beta_y = 1$). Subsistence levels ($\gamma_n, \gamma_q, \gamma_y$) are determined as per the national minimum standards. At least 2 years in primary schooling is set for elementary education while parents are expected to have at least one offspring in their lifetime. The lower bound unit for the consumption basket is set at the poverty line for India and the present value of minimum requirements is estimated at an average for the rural and urban regions for 2005-06.

Further I assign starting values to the choice variables; expected number of children over a person's lifetime (n_0) is set at half the TFR for 2007 as each individual here is assumed to be part of a couple; years of schooling per child (q_0) is fixed at the national standard for average years of schooling and expected lifetime consumption (y_0) can be calculated residually from balancing the budget with discounted present value of lifetime income; the shadow prices ($\pi_{n0} = P_n + P_{nq} q_0, \pi_{q0} = P_q + P_{nq} n_0$) and real income ($R_0 = I + P_{nq} n_0 q_0$) are evaluated at the respective price and income values generated above. Table 1 shows the parameters and variables that are calibrated for the static model simulation exercise.

Calibration Specification (*Normalising* P_y=1)						
Starting Values	n_0	q_0	y_0	R_0	π_{n0}	π_{q0}
	1.371	5.1	514835.9805	524488.615	6081.506	257.82586
Common Parameters	δ	r	I	P_n	P_q	P_{nq}
	0.9357	0.068729	523568.04	5410.04	77.32	131.66
Cobb Douglas Specific Parameters	β_n	β_q	β_y			
	0.015896903	0.002507036	0.981596102			
Leontief Specific Parameters	β_n	β_q	β_y			
	0.000002662984014	0.000009905902166	1			
Stone Geary Specific Parameters	β_n	β_q	β_y	γ_n	γ_q	γ_y
	0.004557875286	0.001614602234	0.993827565	1	2	22871.63
	0.010635235	0.001604744786	0.987760062	0.5	2	22871.63

Table 1: Parameter Calibration results

6.3 Results

The simulation results of the different functional forms of the Q-Q model can be used to see the effects of potential policy experiments and price and income fluctuations. Policies that impact fertility via the Q-Q tradeoff filter into the model via their effect on nominal prices, nominal income, shadow prices and real income. Each policy could had cross program effects that may trickle down further in the system. I first examine how varying the cost of child necessities, schooling expenses and income changes affects an individuals choice of child quality and child quantity directly. After gauging the impact of possible policy instruments affecting the household's decision variables, I then estimate the own price elasticity, cross price elasticity and income elasticity for fertility and schooling with each functional form and interpret the differences arising from the underlying behavioral assumptions.

6.3.1 Comparative Statics

Variation in prices (P_{nq},P_n) and income (I) could affect the household's decision regarding the number of children and the educational investment made in children. The simulations results for quantity of children, level of schooling, the shadow prices and real income with the signs of the derivatives are summarized under Table 2.

The slopes or the derivatives indicate the direction of change that would result if policies were put in place to affect the prices or income faced by a decision making individual from a developing country household. All the partials share the same signs across different functional forms of utility except $\partial lnq/\partial lnP_n$ which differs for the Leontief case as here goods are consumed in fixed proportions so an increase in price reduces consumption of all items equi-proportionately.

The relationships indicate that raising P_n makes consumption requirements of children more expensive which reduces the number of children, increases the shadow price of quantity but lowers the shadow price of quality as well as real income. On the other hand, increasing P_{nq} makes education costlier and has a negative impact on both the number of children and investments in their quality with a rise in the shadow price of quantity, the shadow price of quality and real income. Finally, an increase in nominal full income I causes increments in both quantity and quality as the budget constraint gets relaxed. The only exception is the impact of raising P_n on quality levels of children where in the Cobb Douglas and Stone Geary cases, as child consumption becomes costlier parents have fewer children and allocate the resultant excess income to child quality; but for the Leontief case as all goods must be consumed in the same proportion so an increase in P_n makes quantity of children more expensive which reduces all demands for quantity of children, quality of children and other non-child consumables.

Since my target variable is fertility, I am interested in seeing how the change in P_n and P_{nq} affects the household's choice of n. Raising P_n makes child quantity more costly and this follows the usual trend that higher price leads to lower quantity demanded. However, an interesting finding is that if P_{nq} decreases then fertility may not decline. There is a tradeoff between the rise in quality of the child and the reduction in parent's own out of pocket cost to raise quality and depending on the net effect, policy initiatives to reduce P_{nq} may not always give the anticipated results. This result implies that subsidizing P_{nq} may raise n or the quantity of children and so using government investments reducing cost of education as the sole instrument will be insufficient in reducing fertility via the Quantity-Quality tradeoff.

Utility Fn. → / ↓ Slopes or Derivatives	**STONE GEARY***	**COBB DOUGLAS**	**LEONTIEF**	*Comments*
$\partial \ln n/\partial \ln P_n$	**(-)**	**(-)**	**(-)**	Same sign (-)
$\partial \ln n/\partial \ln P_{nq}$	**(-)**	**(-)**	**(-)**	Same sign (-)
$\partial \ln n/\partial \ln I$	**(+)**	**(+)**	**(+)**	Same sign (+)
$\partial \ln q/\partial \ln P_n$	**(+)**	**(+)**	**(-)**	Different sign (+/-)
$\partial \ln q/\partial \ln P_{nq}$	**(-)**	**(-)**	**(-)**	Same sign (-)
$\partial \ln q/\partial \ln I$	**(+)**	**(+)**	**(+)**	Same sign (+)
$\partial \ln \pi_n/\partial \ln P_n$	**(+)**	**(+)**	**(+)**	Same sign (+)
$\partial \ln \pi_n/\partial \ln P_{nq}$	**(+)**	**(+)**	**(+)**	Same sign (+)
$\partial \ln \pi_q/\partial \ln P_n$	**(-)**	**(-)**	**(-)**	Same sign (-)
$\partial \ln \pi_q/\partial \ln P_{nq}$	**(+)**	**(+)**	**(+)**	Same sign (+)
$\partial \ln R/\partial \ln P_n$	**(-)**	**(-)**	**(-)**	Same sign (-)
$\partial \ln R/\partial \ln P_{nq}$	**(+)**	**(+)**	**(+)**	Same sign (+)

**Note: Stone Geary case is evaluated for both* (γ_n=0.5) *and* (γ_n=1).

Table 2: Policy Experiments with Static Model Simulations

6.3.2 Price & Income Elasticities

The elasticities values indicate the responsiveness of quantity demanded with a percentage change in price or income. Magnitudes of elasticity differ across functional forms due to the nature of consumption behavior as described by the demand and utility structure. The Cobb-Douglas, Leontief and Linear or Stone Geary are some of the most frequently used functional forms which are all special cases of the Constant Elasticity of Substitution (CES) function. As its name suggests, the CES utility function exhibits a constant elasticity of substitution between quality of children, quantity of children and all other non-child goods and services.

The Stone Geary and Leontief forms represent the two extremes for convex utility functions. If we remove the subsistence requirements then in absence of the lower bounds, the Stone Geary structural form collapses to the standard Cobb Douglas case. Generally the linear expenditure form has perfect substitutability between the factors after subsistence requirements have been met while at the other end the Leontief function is characterized by zero substitutability because goods and services must be used in fixed proportions to maintain the level of utility. Hence even as the price ratios change or the budget constraint relaxes, the ratio of child and non-child goods consumed remain unchanged. The Cobb Douglas function represents a middle ground with imperfect substitutability between (n, q, y).

The values of price and income elasticity coefficients represented in Table 3 can be interpreted as impact of changing nominal prices (P_n, P_{nq}), nominal income (I), shadow prices (π_n, π_q) and real income (R) on quantity or fertility behavior (n) and quality or schooling investment (q). Under usual circumstances, the own price elasticity is greatest for the Stone Geary case followed by the Cobb Douglas scenario and finally the Leontief displays least elasticity. However the Static model constructed here has a Stone Geary function where the subsistence requirement for children $(\gamma_n = 1)$ is close to the national average fertility rate per parent[3] $(n_0 = 1371)$. Hence each individual can meet their desired family size simply by attaining a small amount in excess of the lower bound. This makes the linear form less elastic than it would be otherwise.

Comparing across goods, the own price elasticity of fertility with respect to (P_n) and (π_n) is significantly less than one for the Leontief and Stone Geary cases but close to one for the Cobb Douglas case; on the other hand the cross price coefficients for fertility with respect to schooling (P_{nq}) is close to zero for all three systems. For educational investments, the schooling fees represented by (P_{nq}) is only part of the cost so the magnitude does not include the full own price elasticity of schooling as impact of (P_q) is not captured here. Overall the cross price elasticity of schooling is negative but low, so as (P_n) and (π_n) rises net expenditure on children goes up so less is left over to invest in education or child quality. The income elasticities for the Cobb Douglas and Leontief cases are approximately close to unity which is verified from the standard properties of the utility function but the value is low for the Stone Geary case as any relaxation of the budget gets redistributed among all goods to meet subsistence needs.

[3] TFR=2.742 for India in 2007 but this value is reported for women who constitute one half of a couple hence the TFR is halved to represent the per capita average number of children for each parental unit.

ELASTICITY	Utility Fn. / Magnitude of Elasticity**	STONE GEARY*	COBB DOUGLAS	LEONTIEF
Own Price Elasticity of Fertility w.r.t. Nominal Price	$\partial \ln n / \partial \ln P_n$	-0.272	-0.882	-0.050
Cross Price Elasticity of Fertility w.r.t. Nominal Price	$\partial \ln n / \partial \ln P_{nq}$	-0.053	-0.109	-0.038
Income Elasticity of Fertility w.r.t. Full Money Income	$\partial \ln n / \partial \ln I$	0.286	0.998	0.962
Own Price Elasticity of Fertility w.r.t. Shadow Price	$\partial \ln n / \partial \ln \pi_n$	-0.271	-0.990	-0.052
Cross Price Elasticity of Fertility w.r.t. Shadow Price	$\partial \ln n / \partial \ln \pi_q$	0.000	0.000	-0.039
Income Elasticity of Fertility w.r.t. Real Income	$\partial \ln n / \partial \ln R$	0.287	1.000	0.963
Cross Price Elasticity of Schooling w.r.t. Nominal Price	$\partial \ln q / \partial \ln P_n$	-0.007	0.000	-0.024
Own Price Elasticity of Schooling w.r.t. Nominal Price	$\partial \ln q / \partial \ln P_{nq}$	-0.423	-0.693	-0.012
Income Elasticity of Schooling w.r.t. Full Money Income	$\partial \ln q / \partial \ln I$	0.643	0.998	0.988
Cross Price Elasticity of Schooling w.r.t. Shadow Price	$\partial \ln q / \partial \ln \pi_n$	-0.007	0.000	-0.026
Own Price Elasticity of Schooling w.r.t. Shadow Price	$\partial \ln q / \partial \ln \pi_q$	-0.602	-0.990	-0.013
Income Elasticity of Schooling w.r.t. Real Income	$\partial \ln q / \partial \ln R$	0.644	1.000	0.990

**Note: Estimates for Stone Geary case is evaluated for* (γ_n=1) *but the elasticities are different for* (γ_n=0.5).
***Note: Elasticity magnitudes are rounded at 3 decimal places.*

Table 3: Price & Income Elasticity for Static Model Simulations

The difference in the elasticity magnitudes can be understood better if we interpret them in light of the properties of the utility functions.

Leontief: In this case, the goods (n, q, y) are not substitutes so they must be consumed in fixed proportions. Income elasticity is close to unity hence raising income makes a parent proportionately increase demand of all three items in their consumption basket. All price elasticities are at 5% or lower so a price hike for a good does not greatly affect the demand for the good itself as the net increase in expenses means consumption of all goods must go down and so the price effect is weakened. Hence raising the price of fertility or schooling has a small negative effect on demand because of the low degree of substitutability between quantity and quality.

Cobb Douglas: This is the most commonly used functional form and goods (n, q, y) are imperfect substitutes. The income elasticities are almost unity hence a small rise in income results in a large hike in quantity or quality investments. The own price elasticity of fertility and schooling with respect to the shadow prices is also close to unity but the variation with nominal prices is less sensitive. The cross price elasticities however are very low and almost negligible so a change in price of quality has a minor impact on quantity of children.

Stone Geary: For the linear expenditure system, there is a lower bound $(\gamma_n, \gamma_q, \gamma_y)$ that has to be met. Income elasticities are positive but less than half for fertility and just above half for schooling because relaxing the budget constraint means that extra funds must first be allocated towards meeting the minimum necessities. Own price elasticity of fertility is much lower than that for education because the subsistence level is very close to the normal demand for number of children; this allows greater flexibility in choosing the schooling level as compared to the lower bound for childbearing which is more restrictive. The cross price elasticities for both quantity and quality are close to zero.

The uncharacteristically lower elasticity values in the Stone Geary case can be explained by looking at the subsistence parameters[4]; for $(\gamma_n = 1)$ we have each person having at least one child whereby a couple has two children and this is very close to the national TFR hence leaving little room for adjustment. If $(\gamma_n = 0.5)$ then each parent requires half unit of fertility resulting in the couple being satisfied with one child and rerunning the model with the adjusted $(\beta_n, \beta_q, \beta_y)$ parameters yields higher price and income elasticity values. Hence higher fertility costs raises the expenditure of attaining the lower bound in case of higher threshold requirement leaving very little left over income for spending on non-subsistence items.

6.4 Implications of Findings

The different functional forms are useful in describing various types of households and different families may have different priorities, allocation rules or consumption patterns. Families that always choose some goods in a specific proportion may be classified by Leontief preferences where as households that must meet a minimum threshold of some items to survive will be represented by a Stone Geary utility.

Looking across the row, the elasticity values indicate how different utility functions yield different response mechanisms and demonstrates the heterogeneity in individual preference. Along the column, the values show how a given decision making individual who has a particular utility function is affected by price and income fluctuations while picking their choice variables. The

[4] If (γ_n) is positive then childless couples are not meeting subsistence requirement but they form a very small fraction of the sample and can be ignored for the purposes of this analysis.

partial derivatives for number of children and schooling share the same sign across functional forms but understanding not just the direction but the degree of change in fertility from changing the price structure could be beneficial in informing public policy. For all three systems, Cobb douglas, Leontief and Stone Geary, we find that own price elasticity of fertility exceed the cross price effect so a program targeted directly at fertility will have greater effect than a one focussed on education which indirectly filters into fertility choice. So family planning programs will be more effective in fertility regulation than conditional cash transfer schemes. Following up on the policy implications of the comparative static results, even though educational policy subsidizing schooling may result in higher fertility as a consequence; this impact is fairly small scale as seen from the values of cross price elasticity ranging between 3% to 10%. The signs and magnitudes of elasticity values can be used to inform family planning policies targeting fertility behavior.

Theories on the properties of utility functions suggest that real income elasticity of quantity and quality should be unity for the Cobb Douglas and this is established above ($\partial lnn/\partial lnR = 1$ and $\partial lnq/\partial lnR = 1$). In principal the responsiveness of these goods to money income and real income should vary but ($\frac{I}{R} = 0.998$) hence in practice the difference is negligible. For the Stone Geary case on the other hand, demand for quality (0.64) is more income responsive than demand for quantity (0.28) but in general income elasticities are much lower as compared to the Leontief and Cobb douglas form. This is because even if income rises by the same amount in all three functional cases; percentage rise in disposable is same as net rise in income for the non-subsistence scenarios where as in the Stone Geary case if the budget constraint is relaxed, amount spent for meeting threshold requirements remain fixed hence percentage rise in flexible income is much lower. So there exists some stickiness in consumption, more for prices and less for income, which explains the lower elasticity values in the linear expenditure structure.

From the price elasticity calculations, several interesting inferences can be drawn. Impact of policies such as China's one child restriction which makes multiple children prohibitively costly ($\partial lnn/\partial lnP_n$) with a larger effect on the Cobb Douglas case and smaller influence for the Leontief with the Stone Geary system response at an intermediate level. Systems like conditional cash transfers are designed to lower costs of education and from ($\partial lnn/\partial lnP_{nq}$) we find that effect on fertility may be low but there is difference in responsiveness between the functional forms; educational programs meant to boost investment in human capital as per ($\partial lnq/\partial lnP_{nq}$) may have minor consequences in the Leontief structure but there is some feedback from the other systems. Comparing shadow and nominal price cross effects ($\partial lnq/\partial lnP_n$ and $\partial lnq/\partial ln\pi_n$) on quality or schooling of children, the results are almost identical for each type of utility with negligible adverse impact of raising price of childbearing on educational attainment of the child.

7 Policy Recommendation

The purpose of the study is to find feasible and effective instruments that may be used in policy planning to induce desired fertility behavior. Any coercive and non-voluntary policy that infringes upon individual freedoms with regard to reproductive rights would be considered repressive and exploitative measures are unacceptable. A spectrum of factors may potentially affect the fertility of individuals and policies range from direct financial incentives to sociocultural development mechanisms that ensure effectiveness and ethical justice and at the same time manage the population pressure.

Assuming that undistorted behavior is optimal for the household but related externality makes the choice socially suboptimal, every financial instrument will have some associated costs and depending on the type of policy, dead weight losses are incurred by either the household or the administrating body. Any scheme that induces the household to internalize the cost will alter their choice of quantity or quality and result in a dead weight loss for the family as their first best option is now distorted; also any subsidy payment or enforcement cost will result in additional expenditure from the government budget and cause dead weight loss in terms of public finance.

Two of the commonest policy instruments include either a penalty disincentive scheme or a subsidy incentive program. The use of financial incentive or disincentive will affect the price $(P_n, P_{nq}, \pi_n, \pi_q)$ and income (R, I) structure of the economy and the empirical analysis above is an attempt to replicate such scenarios to explain the variation in demand (n,q) as it reacts to such changes. At first glance, the results may show that an incentive or a disincentive scheme will have a symmetric effect on fertility but closer observation indicates otherwise. The price effect of a policy intervention is identical if income is held constant, however any financial instrument will change the income levels. A penalty on high fertility will create a loss in income while a subsidy for lower child bearing will result in income gain, hence different schemes will not have similar effects and the public responsiveness to potential policies must be tested in advance.

Disincentive schemes may be more effective with quicker response rates but a penalty may change the marginal cost for each child and cause loss of income from paying the fines, fees or bribes if one were to exceed the limit. Behavioral studies show that positive incentives are politically more popular than negative reinforcements; so social programs that make smaller families more appealing could encourage a decrease in fertility over a smaller time horizon. Some other possible micro-level policies deterring fertility involve making child schooling mandatory (perhaps with costs of education to be privately borne by parents) or subsiding women's education (so their opportunity cost of time and wages rise).

The findings from this paper supplemented by arguments from past literature implies that better educated, healthy and financially secure individuals tend to have fewer children as predicted by the Quantity – Quality tradeoff models for fertility. Hence Education, Health and Economic Well-being should all be important areas of focus and policy makers should incorporate this into their decision making process during incentive design and budget allocation. Specifically population regulation via education subsidies alone is inefficient as parents will find children cheaper to raise and may raise childbearing; to reduce fertility via the Q-Q tradeoff we must raise child quality in conjunction with other family planning initiatives as well.

8 Conclusion

Scarcity is the motive power behind most population research including the current analysis and one of the chief social concerns involves allocation of scarce resources among alternative competing claims. Resource constrained economies can reduce their ecological footprint and human impact by three means: changing consumption behavior, population regulation or technological innovation. Using the Q-Q tradeoff entails improving child quality levels in terms of health and education, which will directly raise children's wellbeing and as a byproduct reduce the demand for quantity in the long run; higher quality raises income-earning potential and survival probability and at the same time could generate a stable population with replacement rate fertility.

Over time both population growth rates and fertility rates have been on the decline but the absolute population size is still growing. Even after identifying the problem of overpopulation and the aim of population stabilization, any policy that we implement will need a significant response interval. The articles surveyed use a variety of policy instruments ranging from financial incentives to targeted socio-cultural development and try to ensure effectiveness and ethical justice at the same time. Generally, education either generates awareness of birth control techniques or increases the opportunity cost of time for parents and this is found to deter high fertility. Better health implies that mortality rates (both maternal and child) are lower and this reduces the precautionary demand for children as there is less uncertainty about survival till adulthood while higher income or economic well-being seems to be inversely related to fertility.

Socio-cultural factors play an indispensable role in curtailing population growth and for a more time efficient response rate we should include an incentive or disincentive mechanism where desired fertility behavior is rewarded and the converse is met with negative sanctions. Though incentives or disincentives have different structures, they should essentially aim for the same goal. However disincentives are not looked upon favorably because they do not better the quality of life for people and increase the relative deprivation, so incentive mechanism schemes are more preferable. Another issue that must be considered is the length of the planning horizon since this may affect policy choice (tax or subsidy etc.) as per time efficiency; enforcement of policy depends on comparing the relative effectiveness of the instruments given turnover time of the government in power and responsiveness of potential parents.

Monetary or equivalent benefits and penalties or financial disincentives may encourage families to modify their fertility downwards. Commonly used incentives are tax exemptions or direct cash payments which may be one time or deferred schemes. Studies claim that direct payments are part of the program costs for family planning programs while indirect tax exemptions or pension plans are costs borne by society as a whole in order to attain fertility reduction and this must be kept in mind during policy making and budget allocation.

Much research has already been undertaken to better understand ways to curb high population growth by reducing fertility, and my study contributes to the existing literature in several important ways. First, most of the empirical testing in the past has focused on how a higher number of children may lead to lower investments in child quality, but I look at the reverse direction of causality and investigate how increasing child quality may in the long run reduce the demand for quantity as income-earning potential and the probability of survival to adulthood for children increases. Secondly I solve the Q-Q model for different functional forms and run simulations to show how changes in prices and income affect the households decision to invest in number of offspring and their schooling levels. Finally I investigate the impacts of different policy experiments and test the hypothesis that policy initiatives may not always give the anticipated results as simply subsidizing qualitative improvements in children will not necessarily curtail fertility rates; the reduction in parent's out-of-pocket childcare costs to increase quality in terms of health and education may trigger greater childbearing as children are now cheaper to raise.

9 Limitations & Concerns

The results of the study are confined by the data limitations; since 2011 estimates from the recent Indian Census are not available as yet they are being substituted by 2001 indicators. At the policy implementation level, most of the public funding is diverted to critical areas that need immediate attention and very little is left over for family planning policies. Moreover no matter what plan is employed there will be a significant time lag before we see results because fertility decline is a slow process; this may hamper long term planning and policy execution as every five years a newly elected government may come to power and have a different agenda and outlook.

10 Future Extensions

This paper aims to find cost-effective and time efficient policy interventions that have an impact on the proximate determinants of fertility so we can incentivize smaller family sizes for countries suffering from unsustainably high population growth rates due to high fertility. Once we identify appropriate target variables and effective instruments, the counterpart of the model could be applied to below replacement countries to see how parallel policy instruments may be applied to boost birth rates when fertility is viewed as too low. The current findings may also be extended to other developing nations that are facing similar problems after we account for their geographic location and position in the time path of demographic transition. The policy experiments can further be tested for more advanced forms or specifications of the utility function.

Appendix I

The three Static model systems with different functional forms of utility are:

Cobb Douglas Utility function

6 unknowns: n, q, y, π_n, π_q, R

6 equations:

- $n = \frac{\beta_n R}{\pi_n}$
- $q = \frac{\beta_q R}{\pi_q}$
- $y = \frac{\beta_y R}{\pi_y}$
- $R = I + P_{nq}nq$
- $\pi_n = P_n + P_{nq}q$
- $\pi_q = P_q + P_{nq}n$

10 parameters: δ, r, I, P_n, P_q, P_{nq},P_y ,β_n, β_q, β_y with ($\beta_n + \beta_q + \beta_y = 1$)

Leontief Utility function

6 unknowns: n, q, y, π_n, π_q, R

6 equations:

- $n = \frac{\beta_n R}{\beta_n \pi_n + \beta_q \pi_q + \beta_y P_y}$
- $q = \frac{\beta_q R}{\beta_n \pi_n + \beta_q \pi_q + \beta_y P_y}$
- $y = \frac{\beta_y R}{\beta_n \pi_n + \beta_q \pi_q + \beta_y P_y}$
- $R = I + P_{nq}nq$
- $\pi_n = P_n + P_{nq}q$
- $\pi_q = P_q + P_{nq}n$

10 parameters: δ, r, I, P_n, P_q, P_{nq},P_y ,β_n, β_q, β_y with ($\frac{\beta_n}{n} = \frac{\beta_q}{q} = \frac{\beta_y}{y}$)

Stone Geary Utility function

6 unknowns: n, q, y, π_n, π_q, R

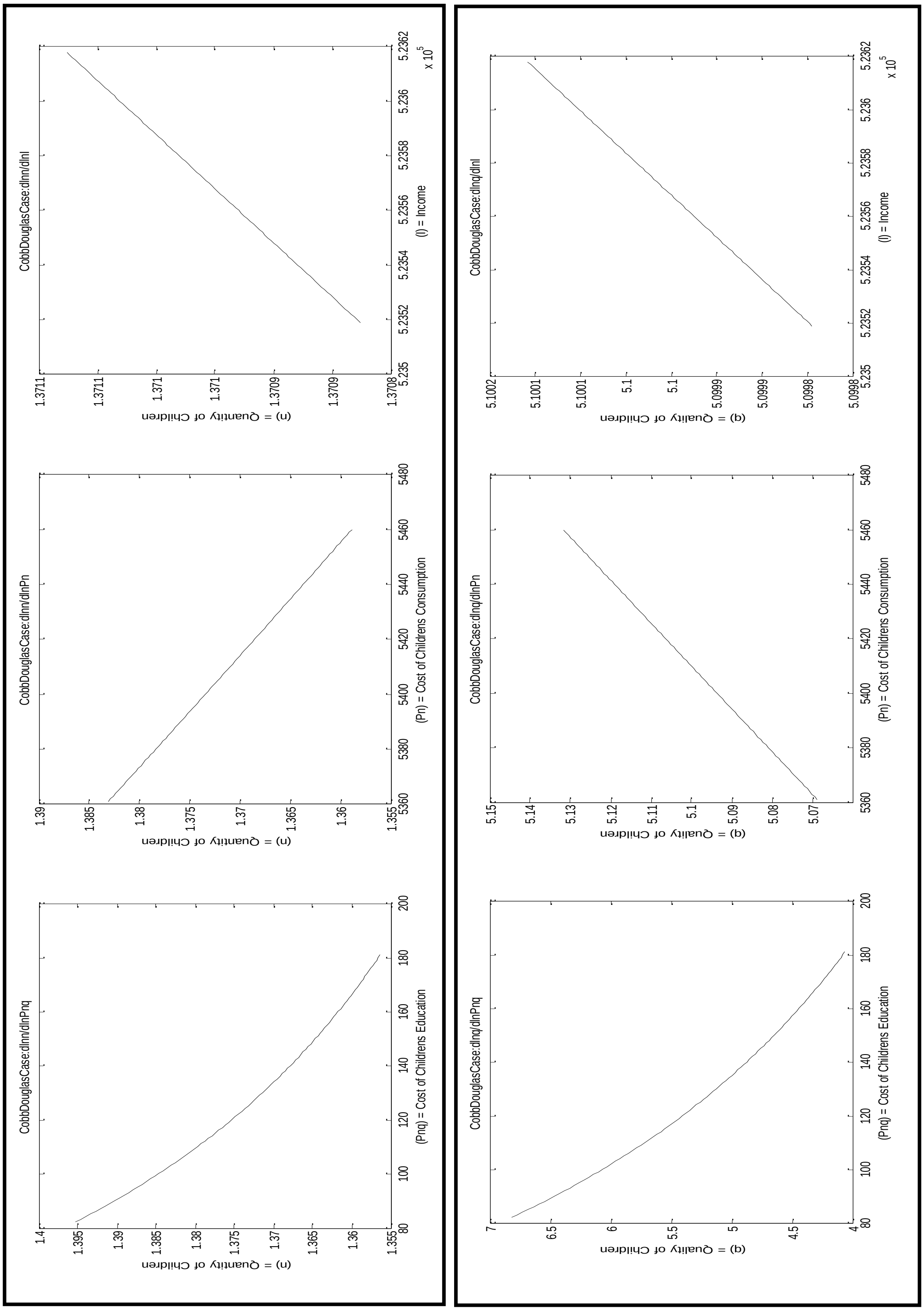
6 equations:

- $n = \gamma_n + (\frac{\beta_n}{\pi_n})[R - \pi_n\gamma_n - \pi_q\gamma_q - P_y\gamma_y]$
- $q = \gamma_q + (\frac{\beta_q}{\pi_q})[R - \pi_n\gamma_n - \pi_q\gamma_q - P_y\gamma_y]$
- $y = \gamma_n + (\frac{\beta_y}{P_y})[R - \pi_n\gamma_n - \pi_q\gamma_q - P_y\gamma_y]$
- $R = I + P_{nq}nq$
- $\pi_n = P_n + P_{nq}q$
- $\pi_q = P_q + P_{nq}n$

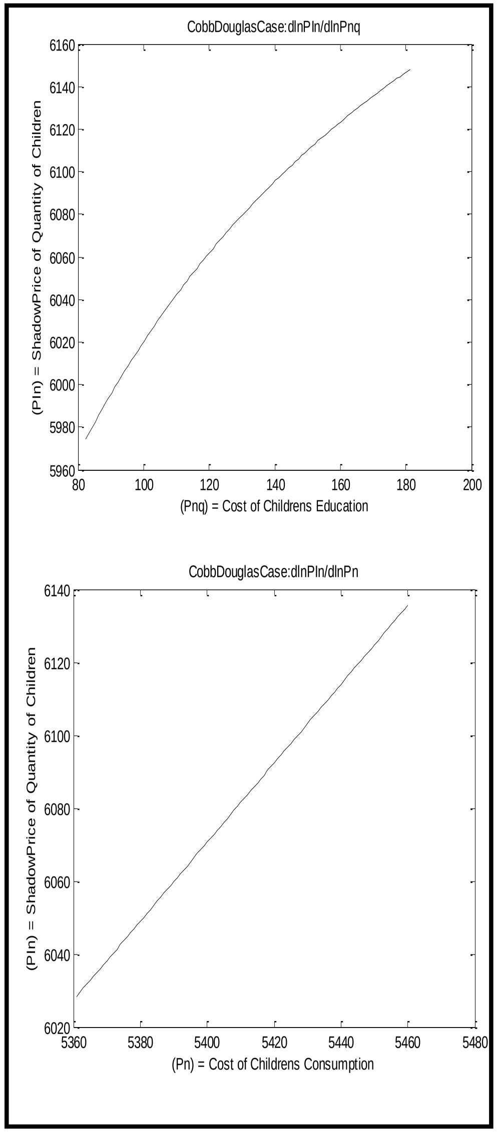
13 parameters: δ, r, I, P_n, P_q, P_{nq},P_y, γ_n, γ_q, γ_y ,β_n, β_q, β_y with ($\beta_n + \beta_q + \beta_y = 1$)

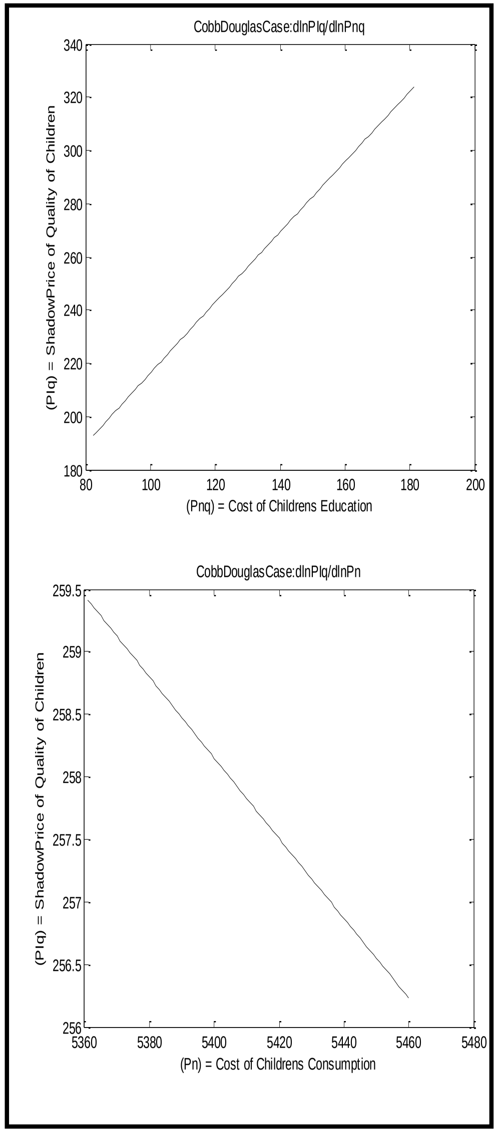
Appendix II

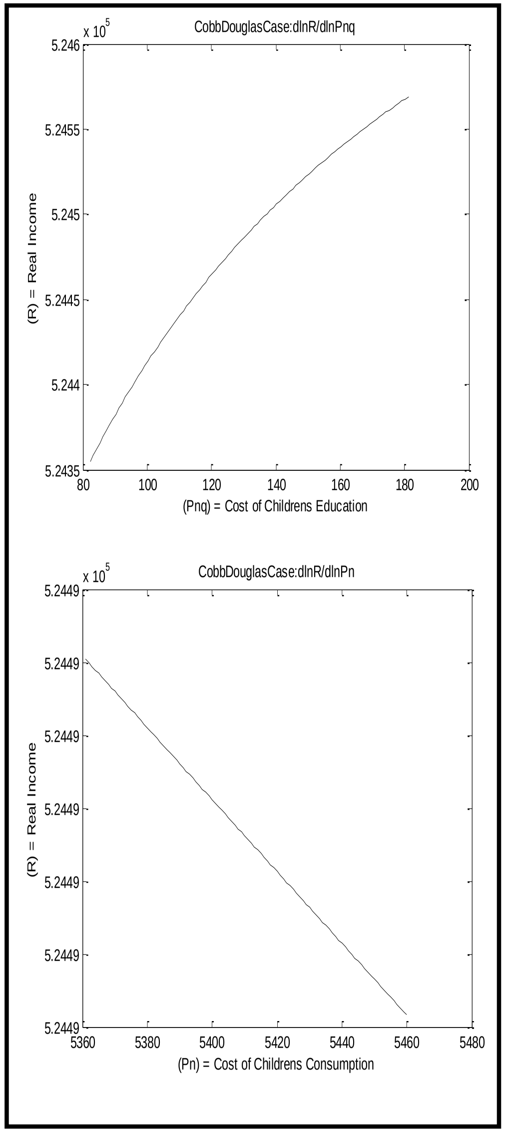
COBB DOUGLAS CASE (β_n=0.015896903;β_q=0.002507036;β_y=0.981596102)



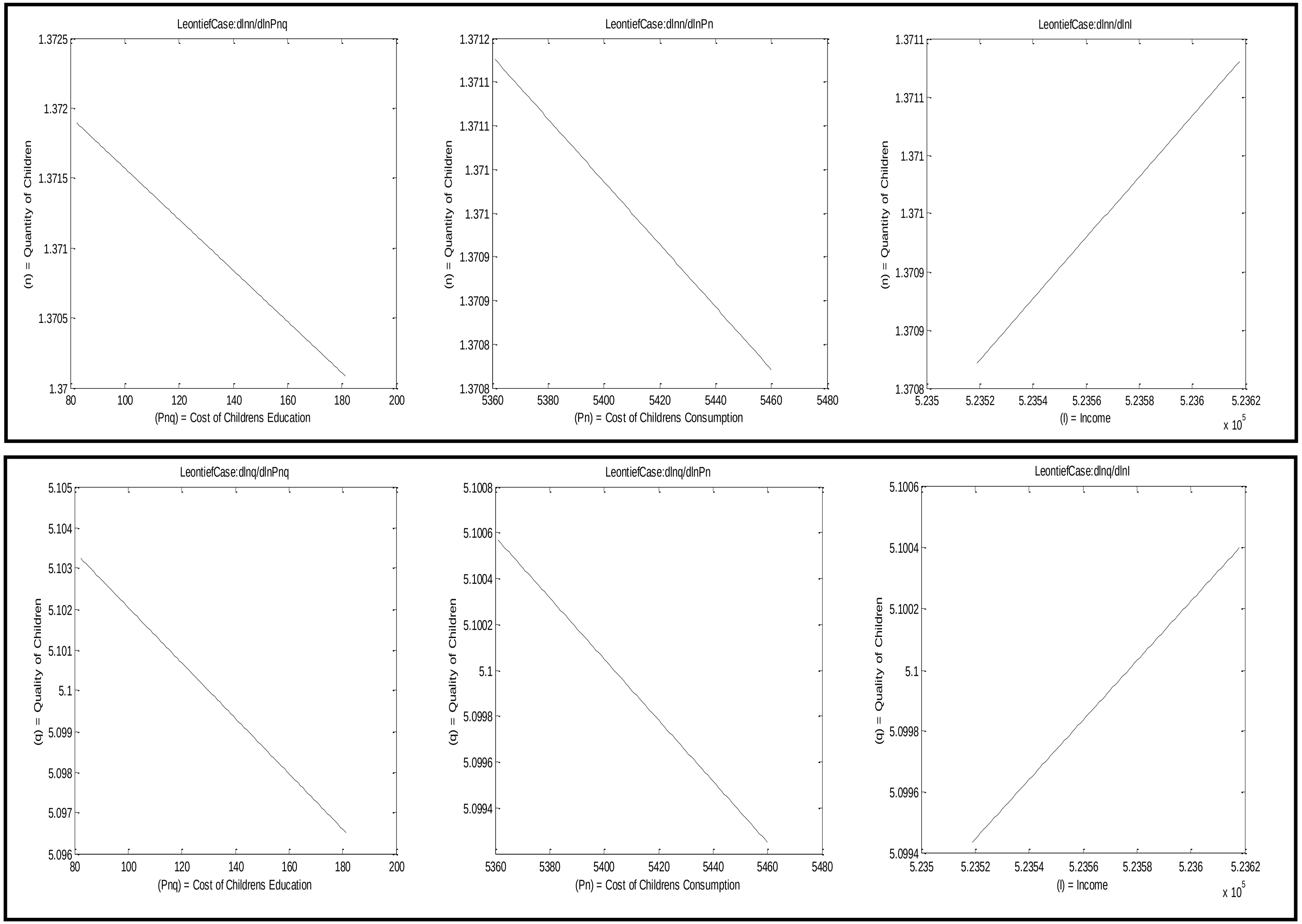
COBB DOUGLAS CASE (β_n=0.015896903;β_q=0.002507036;β_y=0.981596102)







LEONTIEF CASE (β_n=0.000002662984014; β_q=0.000009905902166; β_y=1)



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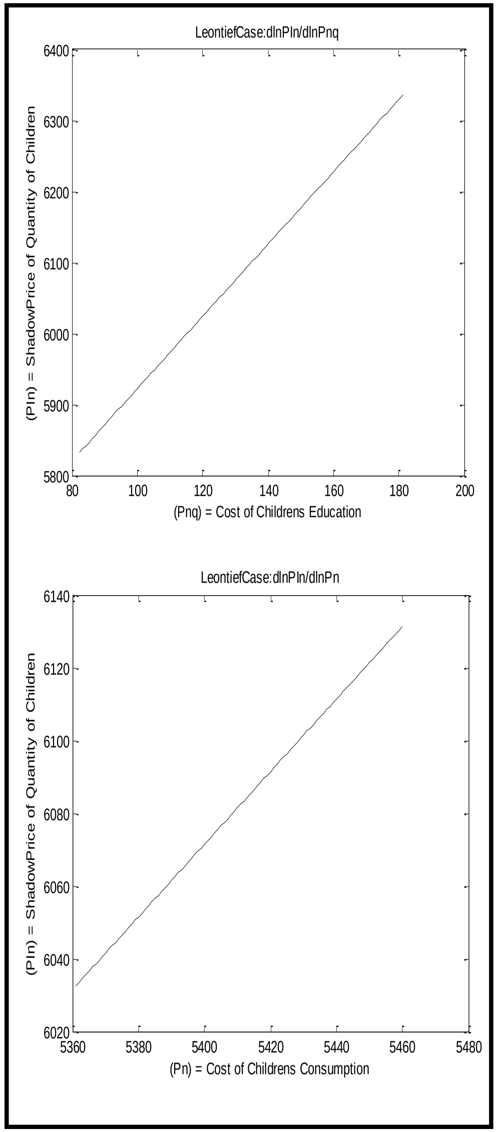
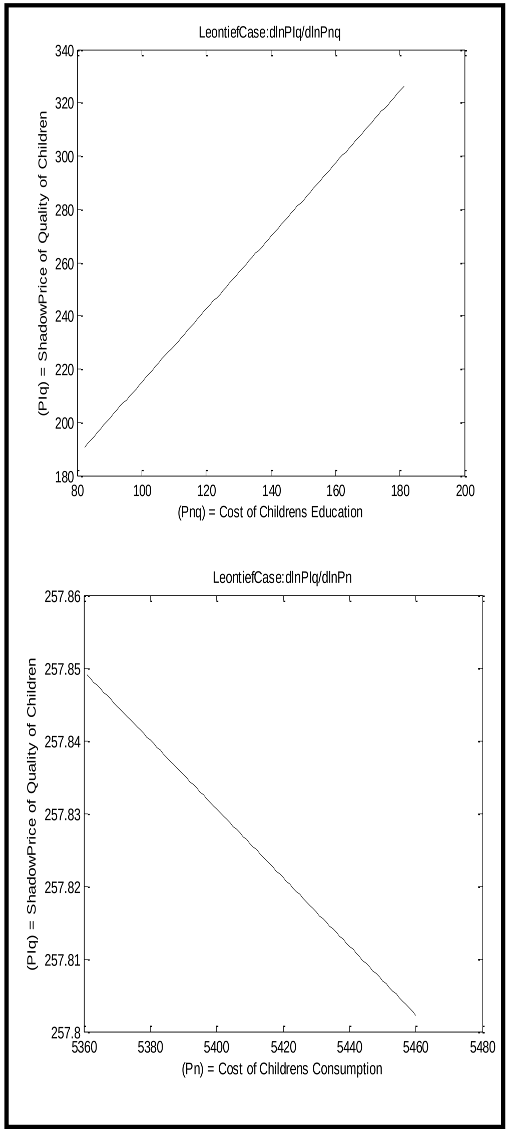
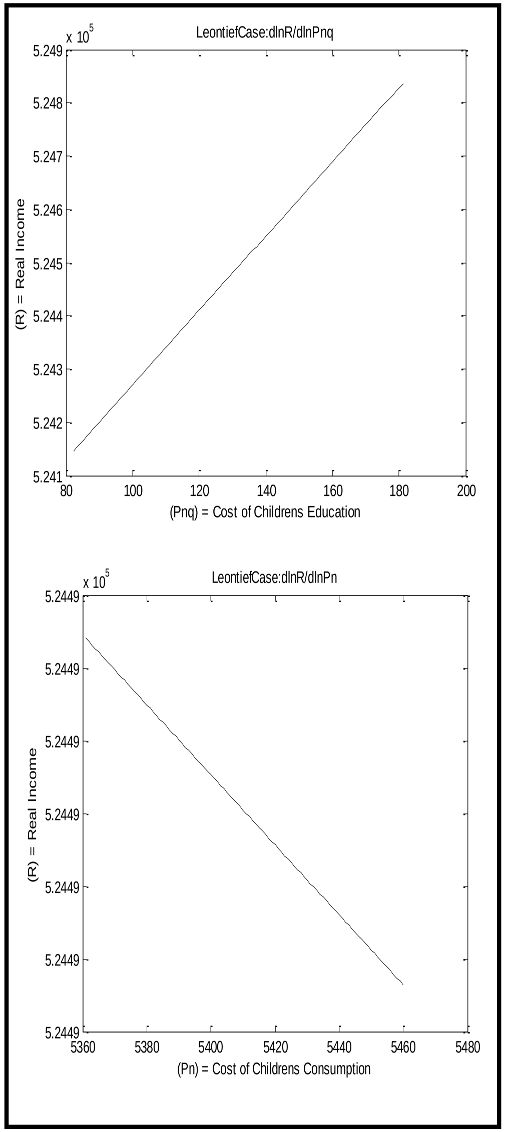
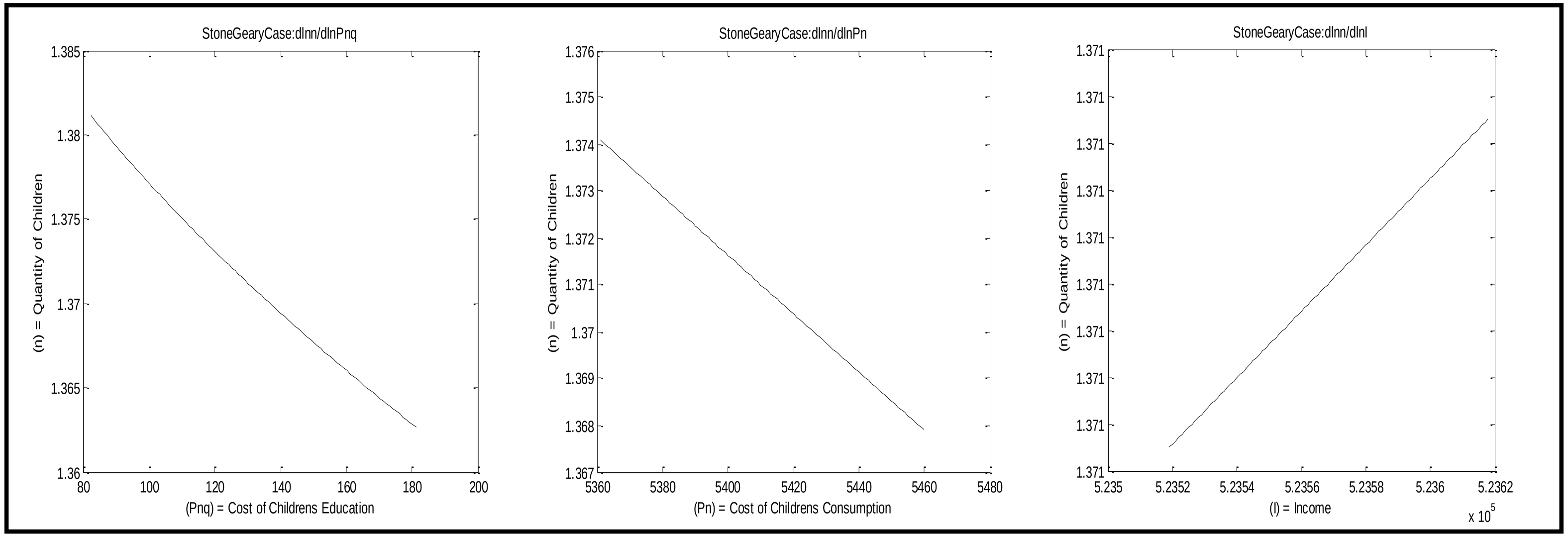
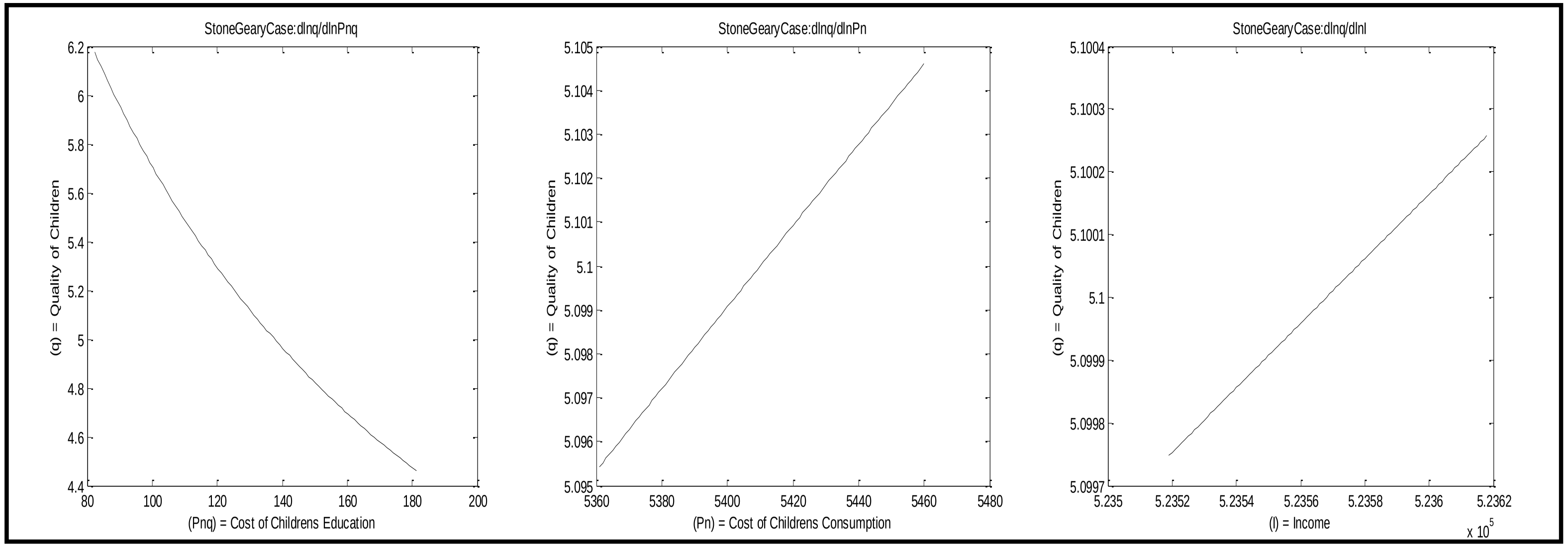




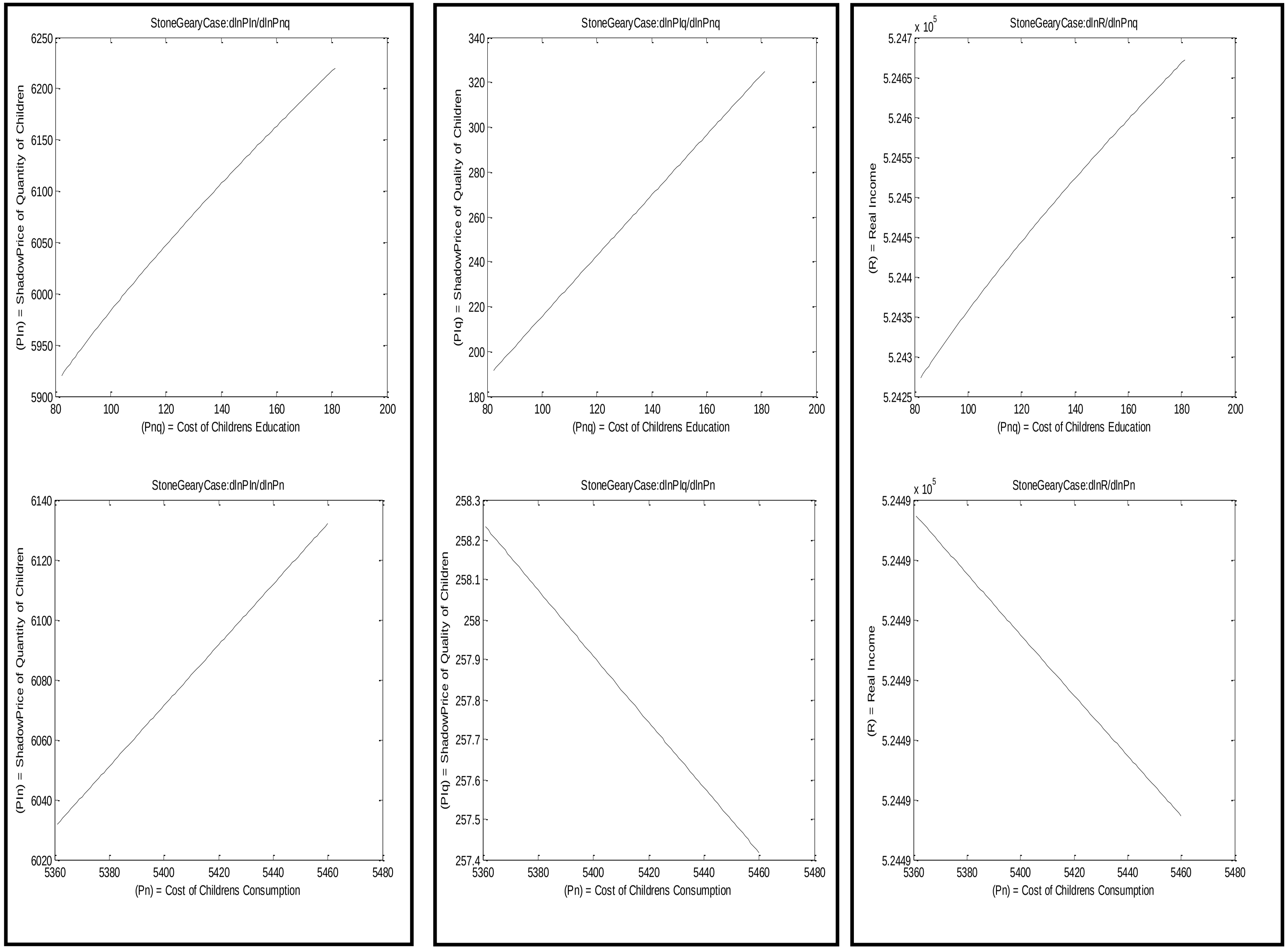


STONE GEARY CASE (β_n=0.004557875286; β_q=0.001614602234; β_y=0.993827565)





STONE GEARY CASE (β_n=0.004557875286; β_q=0.001614602234; β_y=0.993827565)



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