Bayesian cohort component population forecasts

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—DRAFT—

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Abstract

In this paper, we explore the use of Bayesian methods for projecting the United Kingdom's age- and sex-specific population. We first argue that a Bayesian approach is a natural framework for incorporating various forms of uncertainty in probabilistic projections. Second, we demonstrate the consequences of choosing different Lee-Carter type models for fertility, mortality, immigration and emigration in terms of forecasted age patterns and their associated measures of uncertainty. Third, we incorporate these forecasts into a cohort component projection model and compare the results. We end the paper by discussing the merits and flexibility of a Bayesian cohort component projection model and highlight some areas where this work could be extended.

Key words: population projections, Lee-Carter model, uncertainty, United Kingdom, Bayesian

1 INTRODUCTION

In this paper, we explore the use of Bayesian methods for cohort component population projections. The main motivation is the need to incorporate uncertainty into population estimates and projections. Since the 1990s, there has been an increasing

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need to move away from deterministic and variant-style projections to probabilistic projections. Probabilistic projections have the advantage over variant style projections in that they specify the chances or probability that a particular future population value will be within any given range (Alho and Spencer 1985, 2005; Keilmain 1990; Ahlburg and Land 1992; Lee and Tuljapurkar 1994; Lutz 1996; Bongaarts and Bulatao 2000; De Beer 2000). With variant projections, on the other hand, the user has no idea how likely they are, only that they are plausible scenarios representing the 'most likely' and the 'extreme' high and low possibilities. Despite the advantages of probabilistic projections, they have yet to be widely adopted by statistical agencies for several reasons (Lutz and Goldstein 2004). First, there are many types of uncertainties to consider, and including them in projections is not always straightforward, and it can be misleading to include them incorrectly. Second, national statistical offices are often constrained in terms of what methodology they can use for official purposes. Finally, while much has been done, there is still a lot of work needed to produce probabilistic models that are usable at a detailed demographic level, and that are capable of incorporating knowledge of demographic experts.

The rationale for considering a Bayesian approach is that it offers a natural framework to project future populations with uncertainty measures. First, variability in the data and uncertainties in the parameters and model choice can be explicitly included using probability distributions. Second, the approach allows the inclusion of expert judgements, including their uncertainty, into the model framework. Third, the predictive distributions follow directly from the probabilistic model applied. As a result, probabilistic population forecasts, with more reliable and coherent estimates of predictive distributions, can be obtained from a particular projection model.

Demographic events, such as fertility, mortality and migration, tend to exhibit strong regularities in their age patterns. Modelling these age profiles over time permits a relatively concise representation of the history of demographic patterns. These time series can be utilised to allow past trends to be extended to forecast their future behaviour. In this paper, we focus on exploring the consequences of choosing different specifications of age-specific fertility, mortality, immigration and emigration in a cohort projection model in terms of its forecasted populations and measures of uncertainty. For illustration, we use a time series of data from the United Kingdom, consisting of age-specific rates for single years of age going back in time to 1975 for all four components of demographic change. Throughout this paper, we use the term 'forecast' to refer to an outcome of a probabilistic exercise in predictions, as opposed to purely deterministic 'projection' (see, e.g., Keilman 1990).

In the next section of the paper, we provide a general background to forecasting populations, highlighting the issues pertaining to the stochastic modelling of fertility, mortality and migration by age and sex. The data for the United Kingdom (UK) used in this paper to illustrate our approach are described in Section 3. In Section 4, we investigate various ways of specifying Lee-Carter (1992) type models for age-specific fertility, mortality and migration within the Bayesian paradigm. The selected models are then incorporated into the growth matrix of a cohort component projection model. In Section 5, the forecasted populations, with associated measures of uncertainty, are presented and compared, and the merits of the various specifications for including demographic components are evaluated. Finally, in Section 6 the paper is concluded by a general discussion and an outline of further work.

2 BACKGROUND

Concurrent work in Bayesian estimation and projection of populations have been undertaken by Wheldon et al. (forthcoming) and Bryant and Graham (2011; personal communication). Wheldon et al. aim at reconstructing the past population data. Their approach is based on modelling the three population components: fertility, mortality and net migration, as well as account for the varying quality of the population figures available from the censuses. Census data are treated as biased estimates of the true unknown population count. Their approach does not provide a systematic modelling of the age profiles nor do they account for changing behaviours over time. The information about the model parameters is fed into the model in the form of the informative prior distributions. The dynamics in the population are modelled only in the cohort-component model, similar to the one described in Section 4.4 below.

The approach undertaken by Bryant and Graham (2011) envisages a framework for estimating population accounts and projections of the future populations disaggregated by regions, age, sex and time. It combines various data sources which include, among others, vital events registers, censuses, school and electoral rolls. The method relies on the population accounting equation, rather than on the model based on rates, as it is in the model of Wheldon et al. (forthcoming) and the model presented in this paper below. The advantage of the model proposed by Bryant and Graham (2011) consists in combining various data sources into a coherent framework and constraining the true values of population components by the accounting equation. On the other hand, the latter constraint is adding additional complexity to the analysis and computational burden, which can be bypassed by using the rate approach. Also, in the current version age, time, sex and regional patterns are modelled by means of age, time, sex and regional effects and interactions between them (similarly to, e.g., Smith et al. 2010 or Raymer et al. 2011b), within the Poisson-gamma model.

3 DATA

The data underlying our forecasts concern the period 1975-2009. For the forecasting exercise we require data on mortality, fertility, emigration and immigration, disaggregated by age and sex. The data on mortality rates were obtained from the Human Mortality Database (2012). The rates are based on the death counts that originally come from the Office for National Statistics, and exposure-to-risk population, which are Human Mortality Database estimates. The emigration and immigration counts, were obtained directly from the Office for National Statistics. The data on births have been obtained from the Office for National Statistics for England and Wales, as well as from the Northern Ireland Statistics Research Agency and National Records of Scotland. All data are disaggregated by sex and single year of age. The UK mid-year population estimate for 2009, used as a baseline for predictions has been obtained from the Office for National Statistics and Eurostat.

Logarithms of single year mortality rates for females and males from 1975 to 2009 are presented in the upper row of Figure 1. As would be expected, we observe that (i) mortality at all ages, and for both sexes, have been decreasing over time, (ii) females have lower mortality than males and (iii) males exhibit considerably higher mortality in the young adult years.

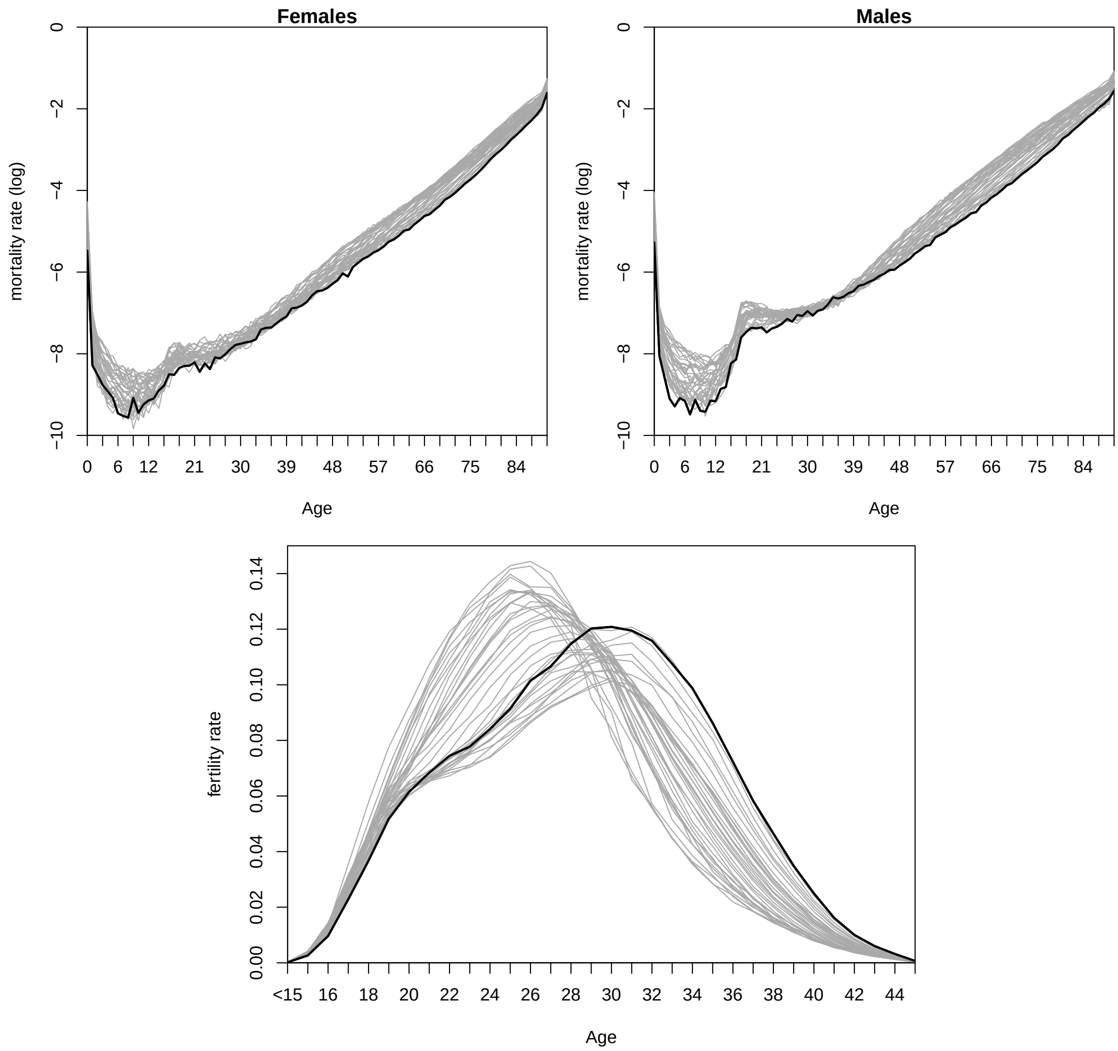

Figure 1: Logarithm of mortality rates and fertility rates in the UK, 1975 to 2009 (black)

Fertility rates by age are presented in bottom row of Figure 1. Over time, we observe a shift from a peak level of fertility in ages 23-26 in 1970 towards one in ages 29-33 years in 2009. The reasons for this shift are related to fertility postponement and a subsequent recuperation. Due to the relatively small counts for very young as well very old ages, the data on births were aggregated into age groups 'under 15 years' and '45+ years'. To compute fertility rates, the same female population-at-risk that was used to calculate the age-specific mortality has been applied, except for the age groups 'under 15' and '45+', where the population-at-risk was aggregated for ages 12-14 years and 45-50 years, respectively.

The total flows of immigration and emigration from 1975 to 2009 are presented in the top row of Figure 2. We observe similar trends in male and female migration over time. The immigration, the levels have been rapidly increasing since the 1990s up until around 2005. For emigration, the increase is less noticeable and there appears to be more volatility, which may be caused by random sample variation in the International Passenger Survey (IPS). Larger irregularities appear when the data are disaggregated single year age groups, as illustrated for immigration and emigration in the middle and bottom rows, respectively, of Figure 2 (see also Raymer et al., 2011).

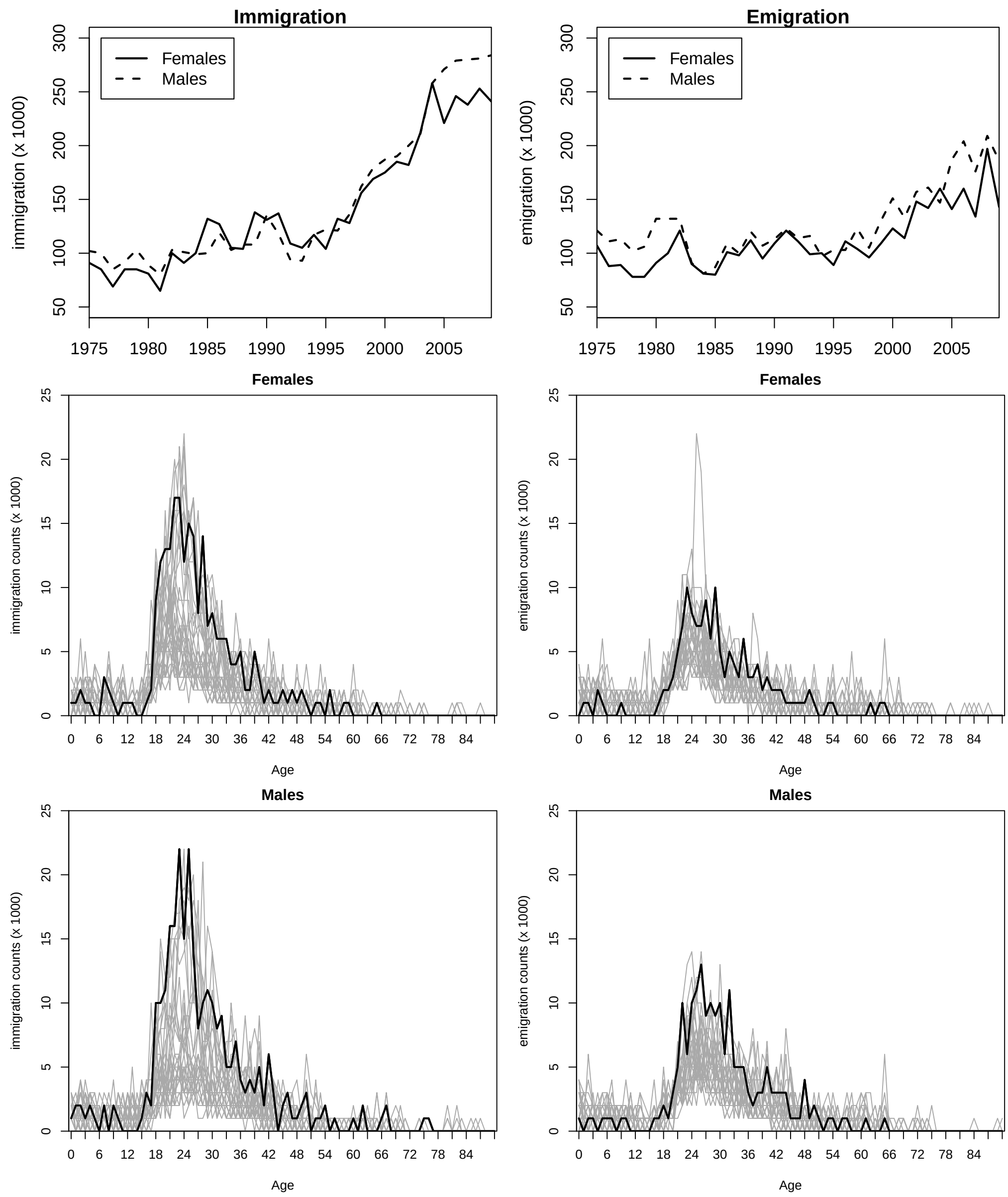

Figure 2: Total levels (tow row) and age profiles of immigration (middle row) and emigration (bottom row) by sex for the United Kingdom, 1975 to 2009 (black)

4 METHODOLOGY

4.1 Forecasting mortality

To forecast the mortality of males M and females F, we consider two models. The first model, denoted by M1, is an extension of the Lee-Carter (1992) approach for age-specific deaths $D_{x,t}$ and population exposed to risk of death $R_{x,t}$, with an assumption that deaths follow a Poisson distribution with mean being the exposure-to-risk $R_{x,t}$ times death rate $\mu_{x,t}$ (Czado et al. 2005). The death rate is log-normally distributed with mean defined as in the original Lee-Carter model, and precision τ. Hence,

$$\begin{aligned} D^k_{x,t} &\sim \text{Poisson}(R^k_{x,t}\mu^k_{x,t}), \\ \log \mu^k_{x,t} &\sim \mathcal{N}\left(\alpha^k_x + \beta^k_x \kappa^k_t, \tau^k\right), \end{aligned} \tag{1}$$

where the additional superscript $k \in \{F, M\}$ denotes females or males. Throughout of this paper, we denote by $\mathcal{N}(\mu, \tau)$ a normal distribution with mean μ and *precision* (inverse variance) τ. The log-normal error term in the equation for mortality rate captures any overdispersion which cannot be explained by the error resulting from the Poisson sampling of deaths. The time-specific parameters κ^k_t for both sexes follow a bivariate VAR(1) process with drift:

$$\begin{pmatrix} \kappa^F_t \\ \kappa^M_t \end{pmatrix} \sim \mathcal{MVN}_2 \left[\begin{pmatrix} \phi_{01} \\ \phi_{02} \end{pmatrix} + \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix} \begin{pmatrix} \kappa^F_{t-1} \\ \kappa^M_{t-1} \end{pmatrix}, \Sigma \right], \tag{2}$$

where Σ denotes a precision matrix.

In comparison with M1, model M2 includes an additional cohort parameter γ_{t-x} (see Renshaw and Haberman, 2006) in the equation for the logarithm of the death rate, that is

$$\begin{aligned} D^k_{x,t} &\sim \text{Poisson}(R^k_{x,t}\mu^k_{x,t}) \\ \log \mu^k_{x,t} &\sim \mathcal{N}\left(\alpha^k_x + \beta^k_x \kappa^k_t + \gamma^k_{t-x}, \tau^k\right). \end{aligned} \tag{3}$$

The model for time-specific parameters κ^k_t is specified as in Equation (2). The cohort effect γ^k_{t-x}, for each sex k, follows a univariate autoregressive process AR(1):

$$\gamma^k_{t-x} \sim \mathcal{N}\left(\psi^k_0 + \psi^k_1 \gamma^k_{t-x-1}, \tau^k_\gamma\right). \tag{4}$$

To ensure identification of the parameters α^k_x, β^k_x, κ^k_t in M1 and M2, as well as γ^k_{t-x} in M2, we impose the following constraints:

$$\sum_{x=0}^{90+} \beta^k_x = 1, \quad \kappa^k_0 = 0, \quad \gamma^k_0 = 0. \tag{5}$$

Since the models are analysed within the Bayesian framework, we need to specify the prior distributions for the model parameters. Thus, for both k we assume

$$\begin{aligned}
&\alpha^k_x \sim \mathcal{N}(0, \tau^k_\alpha), \quad \beta^k_x \sim \mathcal{N}(0, \tau^k_\beta), \quad \text{for all } x, \\
&\tau^k_\alpha \sim \Gamma(2.1, 2.1), \quad \tau^k_\beta \sim \Gamma(2.1, 2.1), \\
&\phi_{ij} \sim \mathcal{N}(0, 1), \quad \psi^k_i \sim \mathcal{N}(0, 1), \quad i = 0, 1, 2, \quad j = 1, 2, \\
&\tau^k \sim \Gamma(0.1, 0.1), \quad \tau^k_\gamma \sim \Gamma(0.1, 0.1), \\
&\Sigma \sim \text{Wishart}\left[\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}, 2\right].
\end{aligned} \tag{6}$$

These prior distributions imply weak information a priori about the model parameters. Hence, they allow the data to 'speak for themselves'. In the specification of the prior distributions for the age-specific parameters α_x and β_x, as well as the choice of the hyper-parameters for the precisions τ_α and τ_β, we follow the suggestions of Czado et al. (2005). However, rather than employing the hyper-parameters based on the data (i.e. the empirical Bayes approach), we use the gamma density, $\Gamma(2.1, 2.1)$, which implies the expected value of precision is one and the standard deviation around 0.7. This allows the estimation algorithm to 'explore' the appropriate region of possible values of the parameter, and also improves the convergence of the algorithm.

4.2 Forecasting fertility

For forecasting age-specific fertility rates, we apply models similar to those for mortality. Here, the age-specific births $B_{x,t}$ are assumed to come from the Poisson distribution with mean being the fertility rate $\nu_{x,t}$ multiplied by the exposure $R^F_{x,t}$, the same as in the model for mortality. As before, we consider a direct extension of the Lee-Carter model, denoted by F1, and the model with an additional cohort parameter, denoted by F2.

Thus, Model F1 is a direct extension of the Lee-Carter model, in which we assume that the time component κ_t follows a univariate autoregressive process AR(1):

$$\begin{aligned} B_{x,t} &\sim \text{Poisson}(R^F_{x,t}\nu_{x,t}), \\ \log \nu_{x,t} &\sim \mathcal{N}\left(\alpha_x + \beta_x \kappa_t, \tau\right), \\ \kappa_t &\sim \mathcal{N}\left(\phi_0 + \phi_1 \kappa_{t-1}, \tau_\kappa\right). \end{aligned} \tag{7}$$

In turn, Model F2 includes the cohort parameter γ_{t-x}, which also follows a univariate autoregressive process AR(1):

$$\begin{aligned} B_{x,t} &\sim \text{Poisson}(R^F_{x,t}\nu_{x,t}), \\ \log \nu_{x,t} &\sim \mathcal{N}\left(\alpha_x + \beta_x \kappa_t + \gamma_{t-x}, \tau\right), \\ \kappa_t &\sim \mathcal{N}\left(\phi_0 + \phi_1 \kappa_{t-1}, \tau_\kappa\right), \\ \gamma_{t-x} &\sim \mathcal{N}\left(\psi_0 + \psi_1 \gamma_{t-x-1}, \tau_\gamma\right). \end{aligned} \tag{8}$$

Again, the identification of the model parameters is achieved by the same set of constraints as in the model for mortality given in Equation (5) for $k = F$. Similarly, the prior distributions for the model parameters are assumed to be the same as in the model for mortality in Equation (6). The only exceptions here concern the univariate distributions for $\phi_j \sim \mathcal{N}(0, 1)$ and $\tau_\kappa \sim \Gamma(0.1, 0.1)$.

4.3 Forecasting immigration counts and emigration rates

Similar to the models for mortality and fertility, the model for immigration counts and emigration rates is based on the Lee and Carter (1992) approach with the Poisson and log-normal extension. Since we acknowledge the fact that emigration and immigration of both males and females can be interrelated, we assume correlation between the time parameters κ_t for all these four components. We also assume that the immigrant counts $I^k_{x,t}$ follow a Poisson distribution with mean $\theta^k_{x,t}$, whereas the Poisson part for the emigrant counts $E^k_{x,t}$ have mean $\eta^k_{x,t}R^k_{x,t}$, which permits modelling and forecasting the emigration rates rather than counts (for a rationale, refer to McDonald and Kippen 2002).

In the model, we also account for the irregularities observed in the data (see Figure 2). Two versions of the model are considered, one without smoothing, denoted by IE1, and the other with smoothing built in into the prior distributions for the age specific model parameters α_x and β_x, denoted by IE2. Unlike fertility and mortality, there is no clear rationale for including a cohort parameter in the model for migration.

Thus, in Model IE1, for both sexes, we assume

$$
\begin{aligned}
E^k_{x,t} &\sim \text{Poisson}(R^k_{x,t}\eta^k_{x,t}), && (9)\\
\log \eta^k_{x,t} &\sim \mathcal{N}\left(\alpha^{Ek}_x + \beta^{Ek}_x \kappa^{Ek}_t, \tau^{Ek}\right), \\
I^k_{x,t} &\sim \text{Poisson}(\theta^k_{x,t}) && (10)\\
\log \theta^k_{x,t} &\sim \mathcal{N}\left(\alpha^{Ik}_x + \beta^{Ik}_x \kappa^{Ik}_t, \tau^{Ik}\right),
\end{aligned}
$$

where superscripts E and I relate to emigration and immigration, respectively. The time-specific parameters follow a multivariate autoregressive process with a drift, as well as logarithmic and linear trends. We simplify the model by assuming that the time parameters κ_t depend only on their own lagged values and not the other direction of flows or the flow for the opposite sex. It is, however, assumed, that time parameters for emigration and immigration for males and females are instantaneously correlated in a given year t. That is, we assume that

$$
\boldsymbol{\kappa}_t \sim \mathcal{MVN}_4\left[\boldsymbol{\phi}_0 + \boldsymbol{\phi}_1 \log(t) + \boldsymbol{\phi}_2 t + \mathbb{I}_4 \boldsymbol{\phi}_3 \boldsymbol{\kappa}_{t-1}, \Sigma\right], \qquad (11)
$$

where $\boldsymbol{\kappa}_t = \left(\kappa^{EF}_t, \kappa^{EM}_t, \kappa^{IF}_t, \kappa^{IM}_t\right)'$, $\boldsymbol{\phi}_i = (\phi_{i1}, \ldots, \phi_{i4})'$, Σ is a matrix of precisions, $\mathbb{I}_4$ is a 4×4 identity matrix, and an apostrophe ($'$) denotes transposition.

In this model, analogous constraints to those for mortality and fertility (see Equation 5) are imposed on the model parameters. The assumed prior distributions for parameters ϕ_{ij}, τ^{ls}, where l denotes the direction of migration (E or I), are the same as in Equation (6). For the remaining model parameters the prior densities are

$$\begin{aligned} &\alpha_x^s \sim \mathcal{N}(0, 0.01), \quad \beta_x^s \sim \mathcal{N}(0, 0.01), \\ &\Sigma \sim \text{Wishart}\left(4\mathbb{I}_4, 4\right). \end{aligned} \tag{12}$$

Model IE2 for emigration and immigration is the same as IE1, except that it includes an algorithm for smoothing the irregularities observed in the International Passenger Survey data. Due to its relatively small sample size and that migrants represent a very small fraction of the overall sample, there are large irregularities observed in the detailed characteristics of migration, such as breakdowns by age and sex (Raymer et al., 2011). These irregularities require smoothing so that the artificial age patterns observed in the data can be avoided. For this purpose, in our model we incorporate smoothing based on the ideas originating from the spatial autoregressive processes, developed by, e.g., Besag (1986).

Having specified the model as in IE1, we construct the prior densities for the age-specific parameters α_x and β_x in the following way. Note, we omit the sex and migration direction superscript for clarity. For the youngest and oldest age groups we assume that the mean of the prior distribution depends on the second youngest and second oldest group, respectively:

$$\alpha_0 \sim \mathcal{N}\left(\alpha_1, \frac{1}{2}\tau_\alpha\right), \quad \alpha_{90+} \sim \mathcal{N}\left(\alpha_{89}, \frac{1}{2}\tau_\alpha\right)$$

and

$$\beta_0 \sim \mathcal{N}\left(\beta_1, \frac{1}{2}\tau_\beta\right), \quad \beta_{90+} \sim \mathcal{N}\left(\beta_{89}, \frac{1}{2}\tau_\beta\right).$$

For the remaining age groups, we assume that their means depend on the average of the two neighbouring age groups $x-1$ and $x+1$:

$$\alpha_x \sim \mathcal{N}\left(\frac{1}{2}\alpha_{x-1} + \frac{1}{2}\alpha_{x+1}, \tau_\alpha\right), \quad x \notin \{0, 90+\},$$

$$\beta_x \sim \mathcal{N}\left(\frac{1}{2}\beta_{x-1} + \frac{1}{2}\beta_{x+1}, \tau_\beta\right), \quad x \notin \{0, 90+\}.$$

The above construction of the conditional precisions for each age group ensures that the unconditional precision is constant for all age groups. Finally, we assume that $\tau_\alpha = \tau_\beta = 0.01$.

4.4 Population projection model

The results of forecasting the four population components of population change are subsequently combined into a cohort component projection model (see Rogers 1995 and Preston et al. 2001). The projection model is specified as

$$\left[\begin{array}{c} \mathbf{P}^F_{t+1} \\ \hdashline \mathbf{P}^M_{t+1} \end{array}\right] = \left[\begin{array}{c:c} a\mathbf{b}_t & \mathbf{0} \\ \mathbf{s}^F_t & \mathbb{O} \\ \hdashline (1-a)\mathbf{b}_t & \mathbf{0} \\ \mathbb{O} & \mathbf{s}^M_t \end{array}\right] \times \left[\begin{array}{c} \mathbf{P}^F_t \\ \hdashline \mathbf{P}^M_t \end{array}\right] + \left[\begin{array}{c} \mathbf{I}^F_t \\ \hdashline \mathbf{I}^M_t \end{array}\right], \tag{13}$$

where $\mathbf{P}^k_t = \left(P^k_{0,t}, \ldots, P^k_{z,t}\right)'$ is vector of mid-year population sizes by age and sex k, $\mathbf{I}^k_t = \left(I^k_{0,t}, \ldots, I^k_{z,t}\right)'$ is a vector of immigration counts. Further, $\mathbf{b}_t = (0, \ldots, b_{14,t}, \ldots, b_{45,t}, \ldots, 0)$ is a vector of birth rates,

$$\mathbf{s}^k_t = \begin{bmatrix} s^k_{0,t} & 0 & 0 & \ldots & & 0 \\ 0 & s^k_{1,t} & 0 & \ldots & & 0 \\ \vdots & & \ddots & & & \vdots \\ 0 & 0 & \ldots & s^k_{z-2,t} & 0 & 0 \\ 0 & 0 & \ldots & 0 & s^k_{z-1,t} & s^k_{z,t} \end{bmatrix}$$

is a matrix of survivorship rates and $a = 1/2.05$ is the assumed proportion of female births in the population. Finally, $\mathbf{0} = (0, \ldots, 0)$ is a vector of length 91 and $\mathbb{O}$ is a matrix of zeros of size (90×91). The survivorship rates come from the mortality and emigration models:

$$s^k_{x,t} = \frac{1 - 0.5(\mu^k_{x,t} + \eta^k_{x,t})}{1 + 0.5(\mu^k_{x+1,t} + \eta^k_{x+1,t})}, \quad \text{for } x \neq z, \tag{14}$$

$$s^k_{z,t} = \frac{1 - 0.5(\mu^k_{z,t} + \eta^k_{z,t})}{1 + 0.5(\mu^k_{z,t} + \eta^k_{z,t})}, \quad \text{for } x = z, \tag{15}$$

and the life-table birth rates come from the fertility model:

$$b_{x,t} = \frac{1}{1 + 0.5\mu^F_{0,t}} \frac{1}{2} \left(\nu_{x,t} + s^k_{x,t}\nu_{x+1,t}\right). \tag{16}$$

An implicit assumption is made about fertility that the rates for boundary ages, that is, 'less than 15' and '45+' are multiplied in the life table by the population aged 14 and 45, respectively. However, since these rates are very small, the overall effect is negligible.

4.5 Model validation and selection

The models for the population components that underlie the population forecast are selected from the models proposed in the previous sections. The selection process is based on (i) visual evaluation of goodness of fit of the model to the data, as well as the produced forecasts, (ii) ex-post evaluation of the in-sample forecasts of the population components based on the 1975-2000 sample, and (iii) the Deviance Information Criterion (DIC) as a formal criterion for model selection.

The DIC (Spiegelhalter et al. 2002) is a tool for assessing the goodness-of-fit of a model to the data, which enables selecting the best performing model. It is often considered a generalisation of the Akaike Information Criterion (AIC) for comparing complex hierarchical models. It utilises a deviance of the likelihood evaluated at the mean of the posterior distribution of the likelihood as the goodness of fit measure, corrected with the so-called 'effective number of parameters' in a model (for definitions and formal derivation, see Spiegelhalter et al. 2002). The requirement for using DIC is that the posterior distribution is approximately multivariate normal. A rule of thumb for selecting the best performing model is similar as for the AIC, namely, the lower the value of the criterion, the better fit of the model.

5 RESULTS

In this section, we present the results of forecasting the population components with the models described in previous section. For each component, we discuss the goodness-of-fit of the model to the data, forecasts of the future patterns and select the underlying model to be used for the population forecast.

5.1 Forecasts of mortality

In the first row of Figure 3, we present the fit of the models M1 and M2 to the 2009 data. It can be observed that the fit of the M2 model with the cohort component reflects the data better than M1. In particular, M2 is able to reproduce the mortality volatility of the cohort born during the pandemic of influenza in 1918-1919. Mortality projected with M1 is lower than with M2, but the age pattern is more uncertain (see second row of Figure 3). This leads to higher but more uncertain predictions of life expectancy (see third row). Predicted life expectancy increases in both M1 and M2, but for the latter model the pace of growth reduces over time.

The underlying model for use in the population forecast is selected by means of combining the formal criteria (DIC), as well as visual inspection of the results and analysis of the in-sample predictions of the model. The visual inspection of the results of both M1 and M2 does not lead to a clear conclusion. It seems that M2, by including the cohort effect, provides a better fit of the model to the data. The importance of the cohort effect in measuring and predicting period mortality rates and the resulting life expectancies is pointed out in recent literature (Luy and Wegner 2009; Luy 2010). In particular, the cohort effects are likely to stem from the long-lasting effects of early-life events and circumstances on mortality, rather than being a result of whole life trajectories experienced by particular cohorts, as demonstrated in a series of longitudinal studies (e.g. Bengtsson and Mineau 2009; see also Murphy 2010 for a general overview and a critical discussion).

To support our rationale, the in-sample prediction of both models on the 1975-2000 sample is analysed. The M1 model yields predictions with relatively large uncertainty, which can be observed in the bottom plot of life expectancy. For M2, the uncertainty is much smaller and, thus, consistent with the results obtained on the

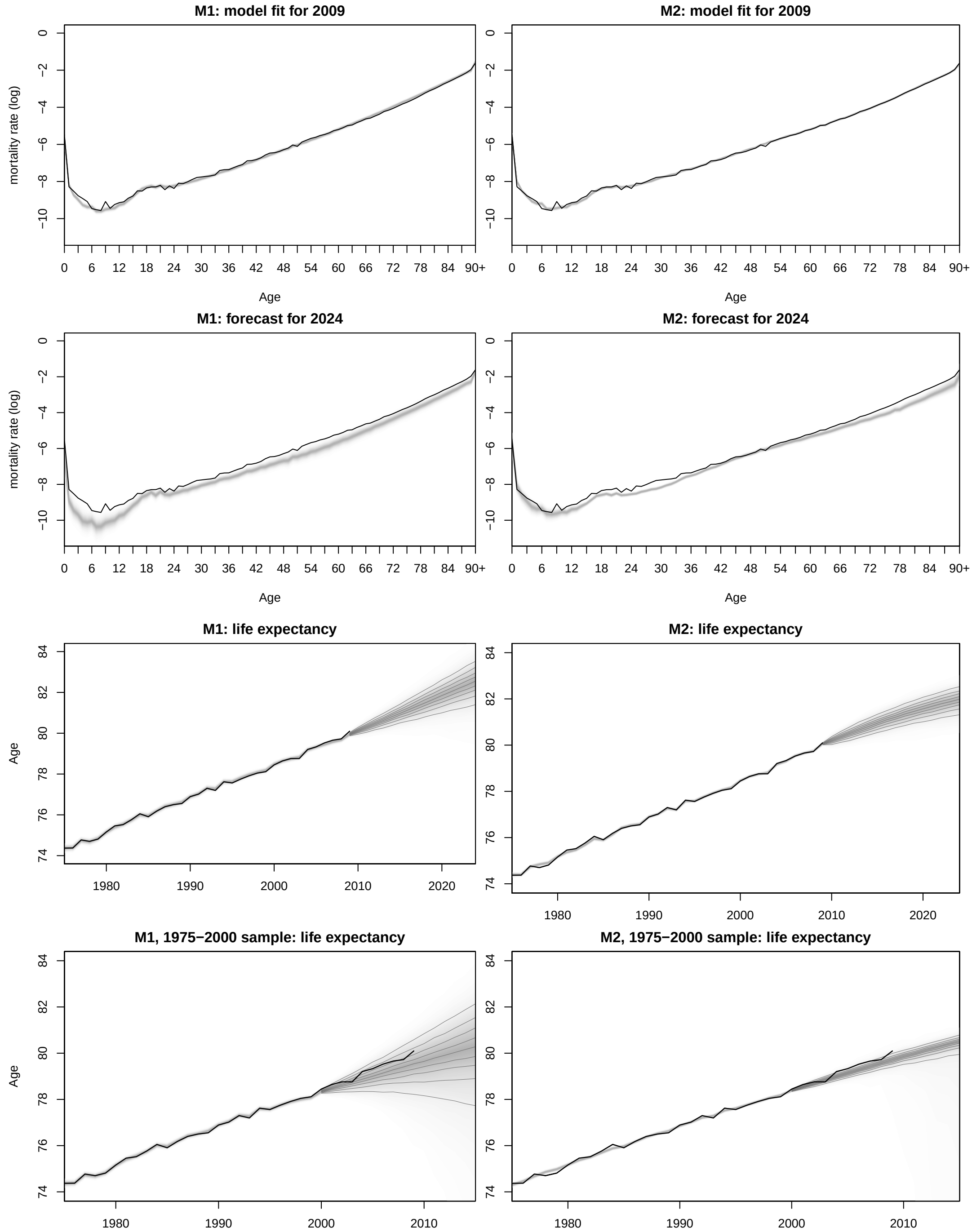

Figure 3: Model fit for mortality to 2009 data for females (top plots), predicted mortality by age for 2024 (second row plots), predicted life expectancy based on the full dataset (third row plots) and 1975-2009 sample (bottom plots).

full sample. Here, the predictions seem to be slightly lower than the observed life expectancy. When we compare the ex-post predictions, it turns out that 59% of the observed mortality rates for years 2001-2009 fall into the 80% predictive intervals in M1 model. For the 95% predictive interval, 80% observations fall into it. The M2 model preforms better; the percentages of the observed mortality rates falling into 80% and 95% predictive intervals are 68% and 86%, respectively. It is also observed that the mortality rates are more often overestimated, which results in a larger decrease in the median forecast of life expectancy than is actually observed. Finally, the DIC points clearly to the M2 model estimated on both full dataset and the truncated sample (see Table 1). It is smaller for the M2 in both full and truncated sample cases.

Table 1: Deviance Information Criteria (DIC) for models for population components

	Mortality	Fertility	Migration
Full model			
Model 1	65970	14790	26100
Model 2	64520	14790	25900
In-sample predictions model			
Model 1	49120	10920	18210
Model 2	48360	10940	18070

5.2 Forecasts of fertility

The age-specific forecasts for fertility are presented in Figure 4. In the first row, we observe the fit of the models F1 and F2 to the 2009 data. The model with the cohort effect (F2) provides a better fit with lower uncertainty. Also, the 2024 forecast (second row) appears more plausible than the forecast based on the F1 model. The F1 model produces an unrealistic median fertility rate of 0.2 for females aged 33-35 years.

The resulting total fertility rate (TFR) is presented in the third row of Figure 4. It is clear from the plots that F2 fits the data better than F1. Moreover, the projected TFR from F1 shows an explosive pattern which we consider unrealistic, with a predicted median TFR of around 2.6 in 2024. Hence, we believe that the pattern of gradual diminishing of the recently increasing TFR produced by F2 reflects our expectations about future fertility in the UK.

The in-sample predictions of the fertility rates confirm our rationale for choosing F2 as the foundation of the population forecast. Again, F2 appears to fit the data better (see fourth row of Figure 4). The resulting forecasts of TFR under F2 seem to be more uncertain than in the case of F1. However, F1 misses the decline in early 2000s. These results are confirmed by the ex-post analysis of the fertility rates. For

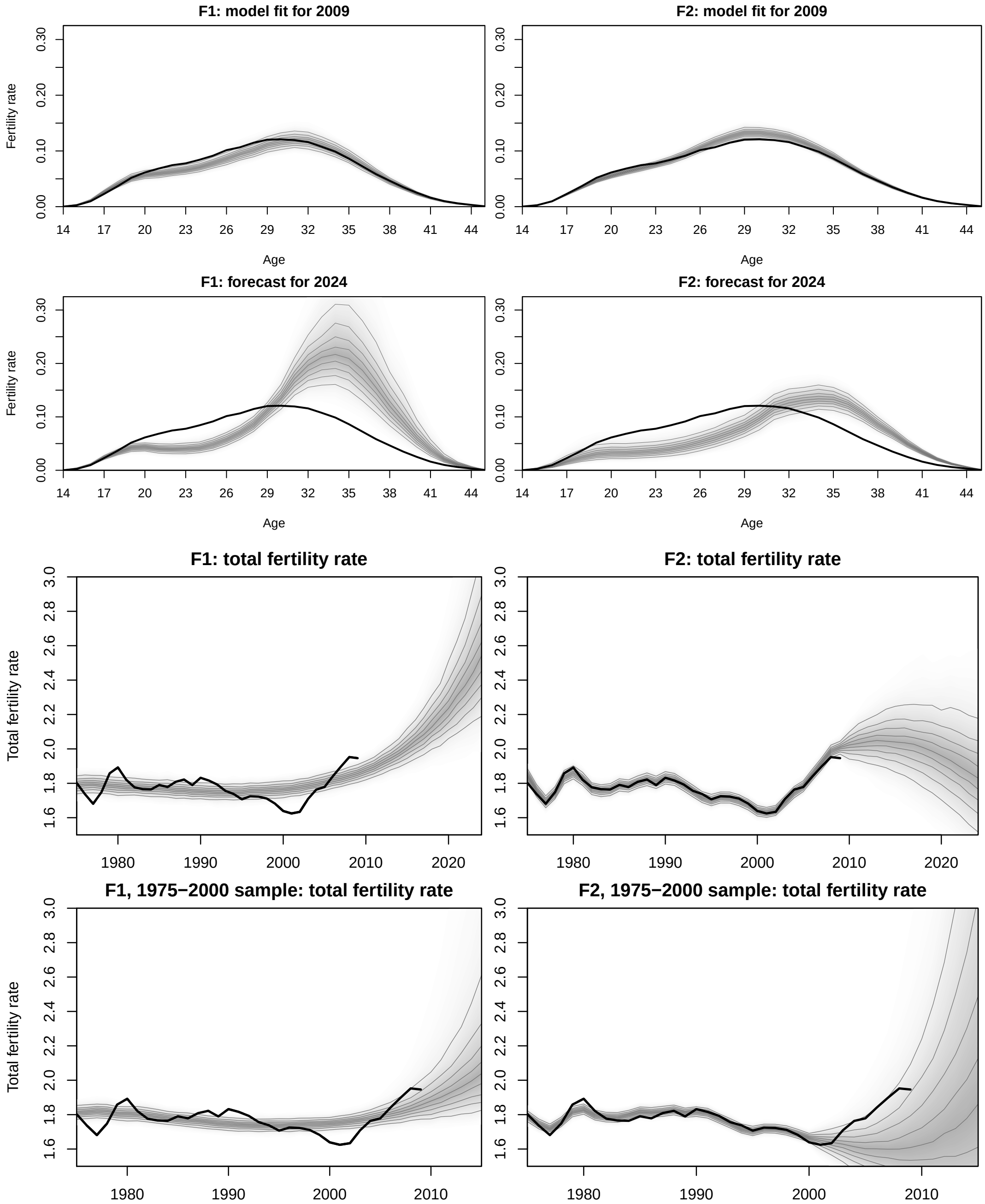

Figure 4: Model fit for fertility to 2009 data for females (top plots), predicted fertility by age for 2024 (second row plots), predicted total fertility rate based on the full dataset (third row plots) and 1975-2009 sample (bottom plots).

F1, 55% of observed fertility rates fall into the 80% predictive interval and 72% fall into the 95% predictive intervals, whereas for F2 the percentage of data falling into respective predictive intervals are 61% and 73%.

The DIC computed for the full sample suggests indifference between F1 and F2, which may be a result of more parameters in F2. When the truncated sample is used to estimate model parameters, F1 obtains a slightly lower DIC. This situation may result from the fact that only 26 observations are used for estimation, which may not be sufficient to capture the cohort effects in the sample. Nevertheless, the ex-post analysis of the in-sample predictions, as well as the visual assessment of the results supports using the model with cohort effect included for the population forecasting. This rationale is supported by vast demographic literature on the quantum and tempo effects in fertility (Bongaarts and Feeney 1998). In particular, we refer to the recent postponement and subsequent recuperation of fertility in many developed countries, where the cohort effects are the most profound (see, e.g., Sobotka et al. 2011). In our results, slightly declining, yet uncertain, fertility rates may indicate yet another period of postponement, which can be possibly linked to difficult economic conditions in the times of budgetary austerity in the United Kingdom in the second decade of the 21st century (Kreyenfeld et al. 2012).

5.3 Forecasts of emigration and immigration

The results of forecasting emigration rates and immigration counts for females are summarised in Figure 5. (Males are not shown for space reasons.) In the first row, we present the IE1 and IE2 forecasts for the year 2009. We observe that both models fit the data reasonably well. As expected, the age patterns yielded by IE2 are smoother than the ones from IE1. The smoothing applied in IE2 also results in reduction of the forecasted emigration rate in 2024, compared to IE1 (first two plots in the second row). The same pattern is observed for males (not shown) and, to a lesser extent, in the predicted age profiles of immigration counts for both males and females. This leads to the differences in the predicted mean emigration rate and total immigration. For instance, the mean total immigration of females in 2024 is 475 thousand for IE1 and 418 thousand for IE2. Smoothing across age also reduced the overall uncertainty of the forecasts.

The results of the in-sample predictions for the mean emigration rate and total immigration in the fourth row of Figure 5 appears to contradict the conclusion from the previous paragraph, especially for the emigration rates. In the example of the predicted mean rate for female emigration, the predictive distribution under IE2 has much heavier tails than under IE1, with median of the migration rate being 0.0035 and 0.004 under IE1 and IE2, respectively. For males (not shown), the difference

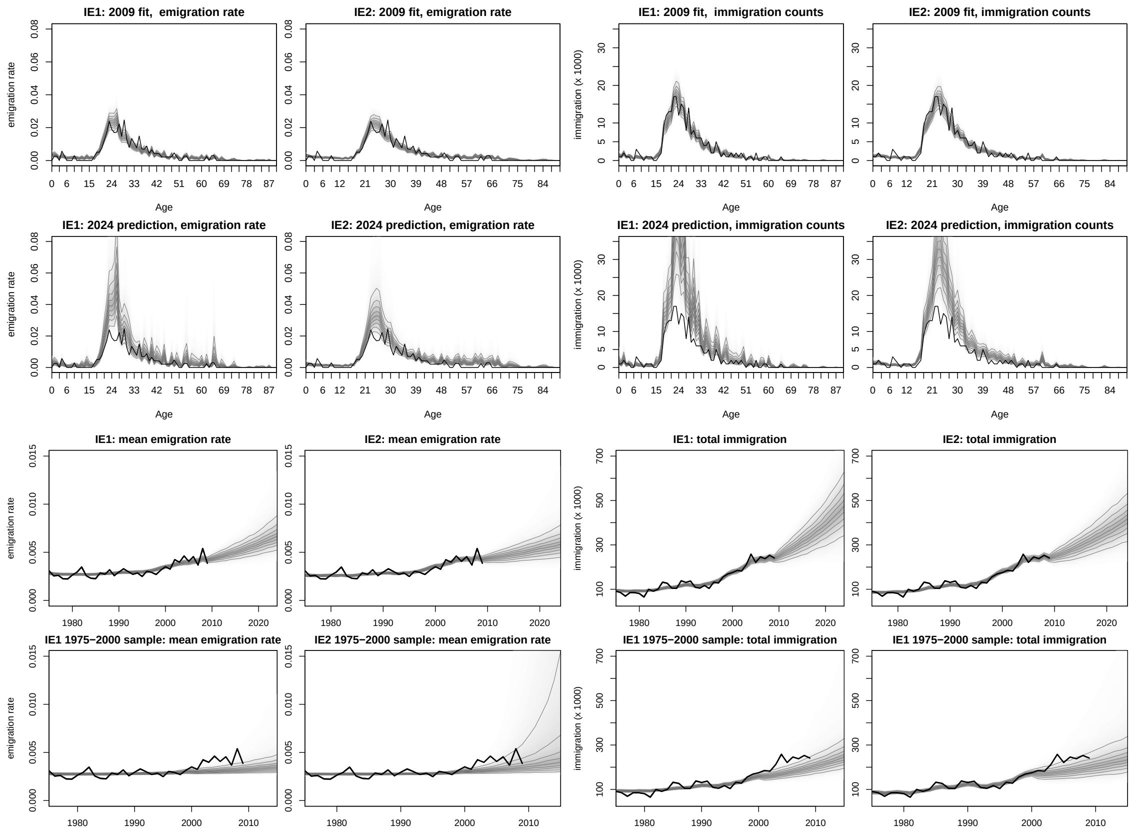

Figure 5: Model fit for immigration counts and emigration rates to 2009 data for females (first row), forecasts for 2024 (second row), predicted mean emigration rate and total immigration based on the full dataset (third row) and 1975-2009 sample (fourth row).

in the means is two-fold. On the other hand, we find that the estimates of the parameters in the model based on the truncated dataset are substantially different from the ones obtained in the full dataset case. In particular, in the truncated data models we find substantial changes in favour of the random walk or explosive behaviour of the emigration rate in the VAR model for the time components of migration; see Equation (11). This situation may result from changes in the observed migration patterns after 2000, especially increase in the volume of immigration and the volatility of emigration.

We believe that the smoothing of the age profiles is necessary in the case of the UK data, due to the irregularities resulting from the small samples (see Section 3). Thus, we select IE2 for our projection exercise, despite its sub-optimal performance in the in-sample prediction exercise. This choice is also suggested by the DIC, which favours IE2 in the analyses of both the full and truncated data (see Table 1).

5.4 Population forecasts

The age composition of the forecasted 2024 population is presented in the first row of Figure 6. Forecasts of the total population for females and males are presented in second row. We observe that the age profile of the 2024 population is shaped mostly by future migration and, to a lesser extent, fertility. The largest uncertainty concerns the youngest population, as well as the population aged 20-45, for both males and females. These findings confirm the observation by Keyfitz (1981) on the plausible limits of population forecasting, which were set to 20 years ahead. It is also expected that the number of the elderly persons will be larger in 2024 but the working-age population, that is, population aged 20-45, will be most numerous.

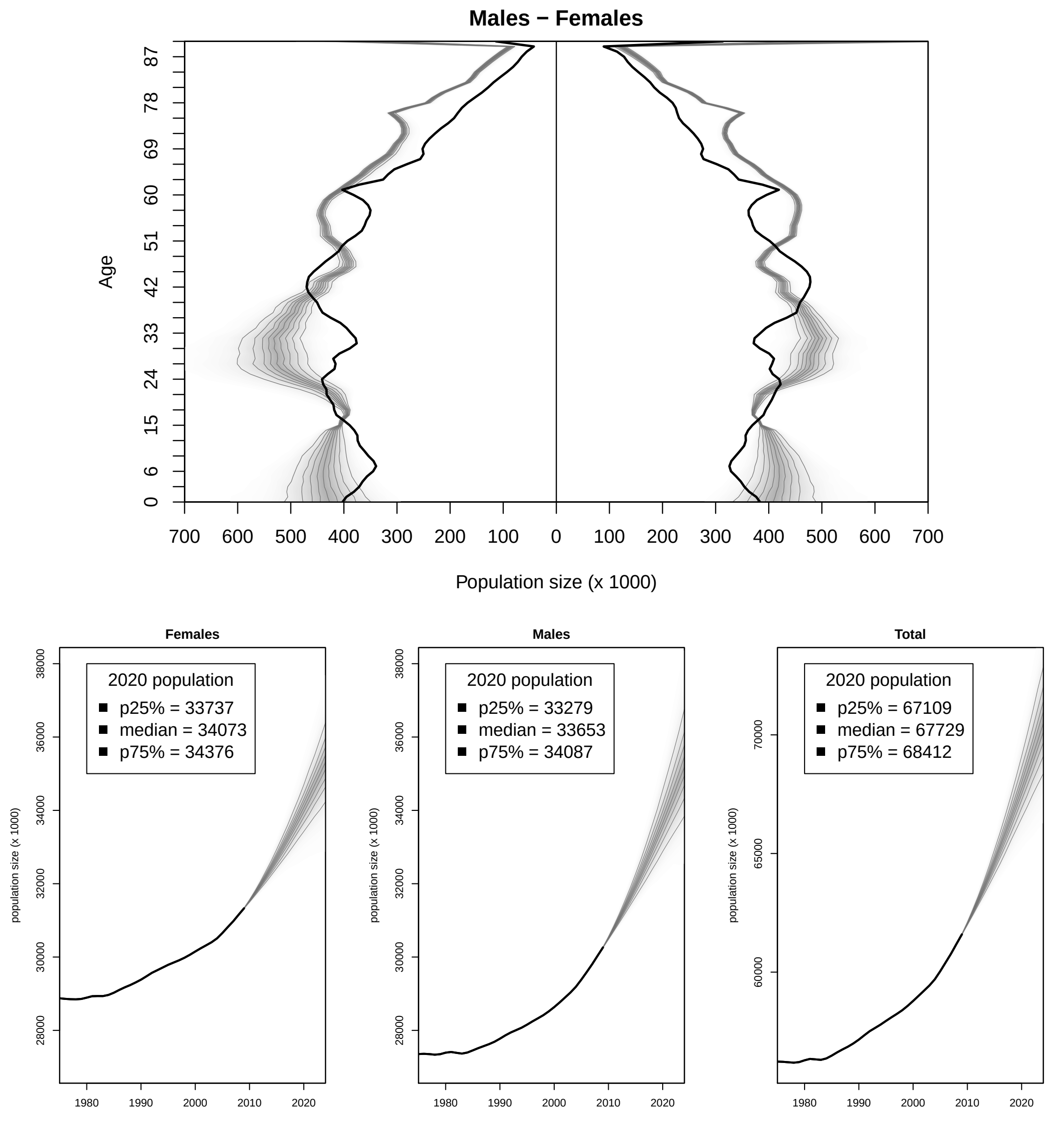

Figure 6: Projected age profiles of males and females (first row) and projected populations of females, males and total (second row).

As far as the total population size is concerned, it is expected that there will be only around 200 thousand more females than males, whereas in 2009 the difference was more than 1 million and in 1975 it was 1.5 million. This is most likely due to the larger proportion of male migration and a gradual closing of the life expectancy gap between the sexes. The median size of the 2024 population is 70.5 million, which is nearly 9 million larger than the population size observed in 2009. For 2020, the forecasted median population is 67.7 million. In the official projections for 2020 prepared by the Office for National Statistics (2011), with 2010 as a baseline population size, the predicted total population is 67.2 million, i.e., half a million lower. However, the Office for National Statistics prediction falls into our 50% predictive interval. The lower Office for National Statistics prediction is a result of a more conservative prediction of international migration. The underlying Office for National Statistics model relies on the assumption that net migration stays constant at the levels observed in recent years, that is at the level of 200 thousand annually (Office for National Statistics 2011).

6 DISCUSSION

In this paper, we treated the Lee-Carter model as a platform for estimating age schedules of the four demographic components of change in the UK. We then combined these components into a single forecast by means of a cohort-component projection model. We also explored the correlation of each of the components in time, as well as between sexes and components (for emigration and immigration), which is embedded in the Lee-Carter method. For emigration and immigration, we provided a tool for smoothing the irregularities in the data.

The main contribution of this paper have been to explore a new approach for integrating demographic components to provide stochastic population forecasts by age and sex. The Bayesian approach that we adopted accounts for the uncertainties embedded in births, deaths, emigration and immigration, as well as across age and sexe. We show that the same general framework of the Lee-Carter approach for modelling age and sex patterns of mortality and fertility can be coherently applied to model corresponding patterns of migration. Irregularities in the data, such as those observed for the UK, can also be accounted for within the model.

Further research should explore other models for forecasting age patterns of demographic components, such as the functional models developed by Hyndman and Booth (2007). Analogously, various specifications for the time component models (such as ARIMA or VAR models of higher order) should be investigated. Next, the underlying models of components for the population forecast can be selected by using various techniques, of which Bayesian model averaging (see Raftery et al 1997) seems to be most appealing. In this way, the model uncertainty would be accounted for in a coherent manner. Finally, the uncertainty of the baseline population size used for projections could be incorporated into the projection. We hope this work provides a foundation for such extensions.

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