

Ultimate fertility levels: a modified projection method for low fertility countries

Leontine Alkema
National University of Singapore

Adrian E. Raftery
University of Washington

Patrick Gerland
United Nations*

*Extended abstract for PAA 2013,
September 20, 2012*

Abstract

Recently, the United Nations Population Division adopted a new method for projecting total fertility (TF) for all countries. The new projection method was well received but raised discussion about the model assumption that in the long run, the TF will oscillate around the approximate replacement level of 2.1 for all countries. In this paper, we investigate a modified TF projection model, whereby the ultimate fertility levels are country-specific and estimated using a Bayesian hierarchical model. Expert opinion is incorporated into the model by setting the upper bound on the ultimate fertility level to 2.1. Under the proposed model, ultimate fertility levels are smaller though within 0.25 child of the current UN projection for most low fertility countries, and 1.9 (80% projection interval 1.6–2.3) for countries that have not yet completed the fertility transition, compared to 2.1 (1.8–2.4) for the existing method.

*Leontine Alkema, Department of Statistics and Applied Probability and Saw Swee Hock School of Public Health, National University of Singapore, Singapore 117546; Email: alkema@nus.edu.sg. Adrian E. Raftery, Department of Statistics, Box 354322, University of Washington, Seattle, WA 98195-4322. Email: raftery@uw.edu. Patrick Gerland, Population Estimates and Projections Section, United Nations Population Division, New York, NY 10017; Email: gerland@un.org.

1 Introduction

The United Nations (UN) publishes projections of the populations of all countries every two years in a publication called the *World Population Prospects* (WPP). It is the only organization to do so. These projections are used by international organizations, governments, particularly those of countries with less developed statistical systems, and researchers. They are used for planning, social and health research, monitoring development goals, and as inputs to other forecasting models.

In the most recent publication, WPP 2010 (United Nations 2011), a new method for projecting total fertility (TF) was incorporated, as described in Alkema et al. (2011). Overall, the new fertility projection method was well received. However, there is some controversy relating to the model assumption in the current method that in the long run the TF will oscillate around the approximate replacement rate of 2.1. This level was deemed too high. For example, Basten et al. (2012) argued that the assumption of an eventual recovery of fertility towards replacement is not justified for five advanced East Asian economies (Korea, Japan, Hong Kong, Singapore and Taiwan). They pointed out that the national statistical agencies of these countries project lower fertility rates than does the UN, that the relevant scientific literature does not suggest an increase in fertility in the short term, that a recent unpublished survey of experts concluded that fertility would not increase as markedly as the UN predicts and that current evidence about fertility intentions does not suggest an immediate appetite for more children in these countries. They also argued that the UN assumption is based on European experience and that there is no reason to assume that it will carry over to East Asia, and that the “low fertility trap” hypothesis may apply to these countries (Lutz 2008).

Here we suggest a possible way to relax the assumptions underlying the fertility model so as to accommodate the critique related to the assumption for ultimate fertility level and make the model more fully data-based. We first summarize the current UN approach in Section 2, followed by the presentation of the modification to the method to accommodate uncertainty about the long-term behavior of fertility. In Section 3 we compare the results of the modified method to the current method.

2 Fertility Projection Model

Three phases model In WPP 2010, TF projections (in 5-year steps) were constructed based on observed trends in the past. The typical evolution of fertility over time in a country was described and modeled in three phases, shown in Figure 1 (Alkema et al. 2011). Phase I precedes the beginning of the fertility transition and is characterized by high fertility that is stable or increasing. Phase II consists of the demographic transition during which fertility declines from high levels to below the replacement level of 2.1 children per woman. Phase III is the post-transition period, characterized by low fertility rates and potential recovery towards or oscillations around replacement-level fertility.

The projection model for Phase III, that determines the ultimate fertility level and is the focus of this paper, is based on trends that have been observed in countries that are currently in Phase III. A country is defined as having entered Phase III once two consecutive five-year

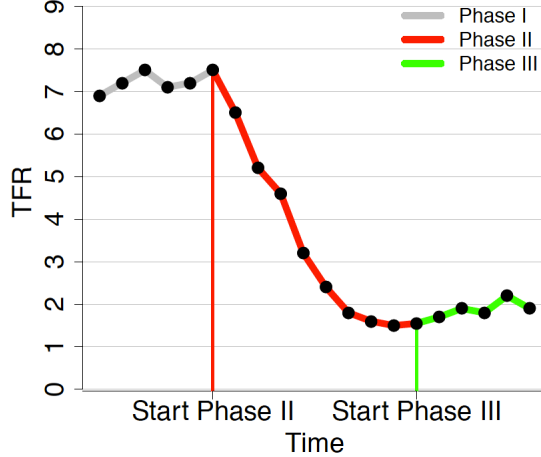


Figure 1: The three phases of the model for total fertility — Phase I: pre-demographic transition; Phase II: demographic transition fertility decline; Phase III; post-transition recovery.

increases below a TF of 2 children have occurred. By this definition 21 countries had entered Phase III by 2010: 19 European countries (Belgium, Bulgaria, Channel Islands, Czech Republic, Denmark, Estonia, Finland, France, Germany, Ireland, Italy, Latvia, Luxembourg, Netherlands, Norway, Russian Federation, Spain, Sweden and the United Kingdom), the USA and Singapore. For these countries, TF has tended to increase back towards replacement level after they entered Phase II, reversing the secular trend of fertility decline. This is by now a well-documented trend, especially in Europe (Myrskylä et al. 2009).

Phase III model incorporated in WPP 2010 In WPP 2010, recovery towards and oscillations around approximate replacement fertility were modeled using a single first-order autoregressive model with long-term mean equal to the approximate replacement fertility level of 2.1 for all countries. This model for TF, denoted by $f_{c,t}$ for country c , 5-year period t , is given by

$$f_{c,t+1} - \mu = \rho(f_{c,t} - \mu) + \varepsilon_{c,t}, \quad (1)$$

$$\varepsilon_{c,t} \stackrel{\text{iid}}{\sim} N(0, \sigma_\varepsilon^2), \quad (2)$$

where long-term mean $\mu = 2.1$, ρ is the autoregressive parameter and σ_ε^2 the variance of the distortion terms. Based on this model, the TF is expected to increase if it is below replacement. The expected increase toward 2.1 is larger if the current TF is farther from 2.1, and it depends on ρ : the smaller ρ , the more quickly the TF will increase toward replacement-level fertility. Maximum likelihood estimation using the 54 time periods that have been observed in the 21 countries that have entered Phase III yielded $\hat{\rho} = 0.89$ and $\hat{\sigma} = 0.10$.

Bayesian hierarchical model The modeling assumptions in Eq. (1) can be relaxed by allowing both μ and ρ to vary between countries:

$$f_{c,t+1} - \mu_c = \rho_c(f_{c,t} - \mu_c) + \varepsilon_{c,t}, \quad (3)$$

where μ_c and ρ_c are the country-specific ultimate fertility level and autoregressive parameter, respectively. Estimating μ_c and ρ_c for a single Phase III country is challenging because the data are sparse (based on WPP 2010, at most six consecutive five-year periods have been observed in a single country, see Discussion). Therefore, we model the values of the parameters as arising from a “world” distribution. This leads to estimates that borrow strength from data for other countries, and makes the model hierarchical. The hierarchical model for μ_c is given by:

$$\mu_c \sim TN_{[0,\infty)}(\bar{\mu}, \sigma_\mu^2), \quad (4)$$

where $\bar{\mu}$ represents the world mean parameter for the country-specific asymptotes, σ_μ^2 their variance, and $TN_{[a,b]}(e, f^2)$ denotes a truncated normal distribution with mean parameter e and standard deviation parameter f , truncated to lie between a and b , such that μ_c is restricted to be positive. For the autoregressive parameter

$$\rho_c \sim TN_{[0,1]}(\bar{\rho}, \sigma_\rho^2), \quad (5)$$

such that ρ_c is restricted to be between 0 and 1 to guarantee stationarity of the time series process (bounded projection intervals).

Estimating the world level parameters $\bar{\mu}$, σ_μ , $\bar{\rho}$ and σ_ρ in a Bayesian framework allows for easy incorporation of expert opinion. The following priors are used:

$$\bar{\mu} \sim U[0, 2.1]; \quad \sigma_\mu \sim U[0, 0.318]; \quad \bar{\rho} \sim U[0, 1]; \quad \sigma_\rho \sim U[0, 0.289]. \quad (6)$$

The prior on $\bar{\mu}$, the world mean parameter for the country-specific asymptotes, is restricted to be no greater than the replacement level of 2.1, in line with expert opinion. The upper bounds on σ_μ and σ_ρ are based on the assumption that the spread in country-specific parameters is at most comparable to the spread of random draws between 1 and 2.1 for μ_c , and 0 and 1 for ρ_c respectively.

Markov chain Monte Carlo methods were used to obtain 3,000 samples from the posterior distribution of all the model parameters. Based on posterior sample $j = 1, \dots, 3000$, future TF values for a country that is currently in Phase III are sampled as follows:

$$f_{c,t+1}^j \sim N(\mu_c^j + \rho_c^j(f_{c,t}^j - \mu_c^j), (\sigma_\varepsilon^j)^2), \quad (7)$$

where μ_c^j , ρ_c^j and σ_ε^j refer to the j -th sample from the respective posterior distributions. To construct Phase III projections for countries that will enter the post-transition phase in the future (for which no data has been observed yet), the j -th sample μ_c^j and ρ_c^j are drawn as follows:

$$\mu_c^j \sim TN_{[0,\infty)}(\bar{\mu}^j, (\sigma_\mu^j)^2); \quad \rho_c^j \sim TN_{[0,1]}(\bar{\rho}^j, (\sigma_\rho^j)^2), \quad (8)$$

where $\bar{\mu}^j$, σ_μ^j , $\bar{\rho}^j$ and σ_ρ^j refer to the j -th sample of the hierarchical parameters.

3 Results

Country projections TF projections based on the WPP 2010 and Bayesian hierarchical model for all 21 countries that are currently in Phase III are given in Figure 2. For most country-periods, the median projections from the hierarchical model are below the projections using the WPP 2010 method. For all but three countries (Channel Islands, Germany and Singapore) the absolute difference in the projection for 2095–2100 is less than 0.25 child. For all but one of the 21 countries, the WPP 2010 projections are within the 80% projection intervals (PIs) as constructed based on the hierarchical model. The exception is Singapore, which is also the only one of Basten et al. (2012)’s five advanced East Asian economies that has entered Phase III. The projection under the Bayesian hierarchical model is much lower than under the WPP 2010 model, as Basten et al argued it should be. It asymptotes at 1.5 instead of 2.1. This suggests that the data provide some support for Basten et al’s contention for Singapore, and also that the Bayesian hierarchical model can accommodate differences of this kind between countries.

Future fertility levels for the ‘not-yet-in-Phase-III’ countries For countries that are projected to enter the post-transition phase in the future, we find that the hierarchical model provides projection intervals that slightly lower and wider than those based on the current UN method. The Phase III projection is illustrated in Figure 3 (left plot) for a starting level of 1.4 in the period 2005–2010 (centered in 2008). The projection intervals from the WPP 2010 method are also shown for comparison. The projected TF under the hierarchical model is 1.9 for 2095–2100 (with 80% PI ranging from 1.5 to 2.2), compared to 2.0 (1.7–2.3) for the WPP 2010 method.

As the projection horizon increases, the distribution of TF values in a given period converges to its ultimate/asymptotic distribution. Figure 3 (right plot) illustrates the asymptotic distribution for the TF with the WPP 2010 method (blue) and the proposed hierarchical method (red), with 80% PIs (horizontal lines). The 80% PIs are given by (1.8, 2.4) for the WPP 2010 method and (1.6, 2.3) for the proposed method. The posterior medians are 2.1 and 1.9 respectively.

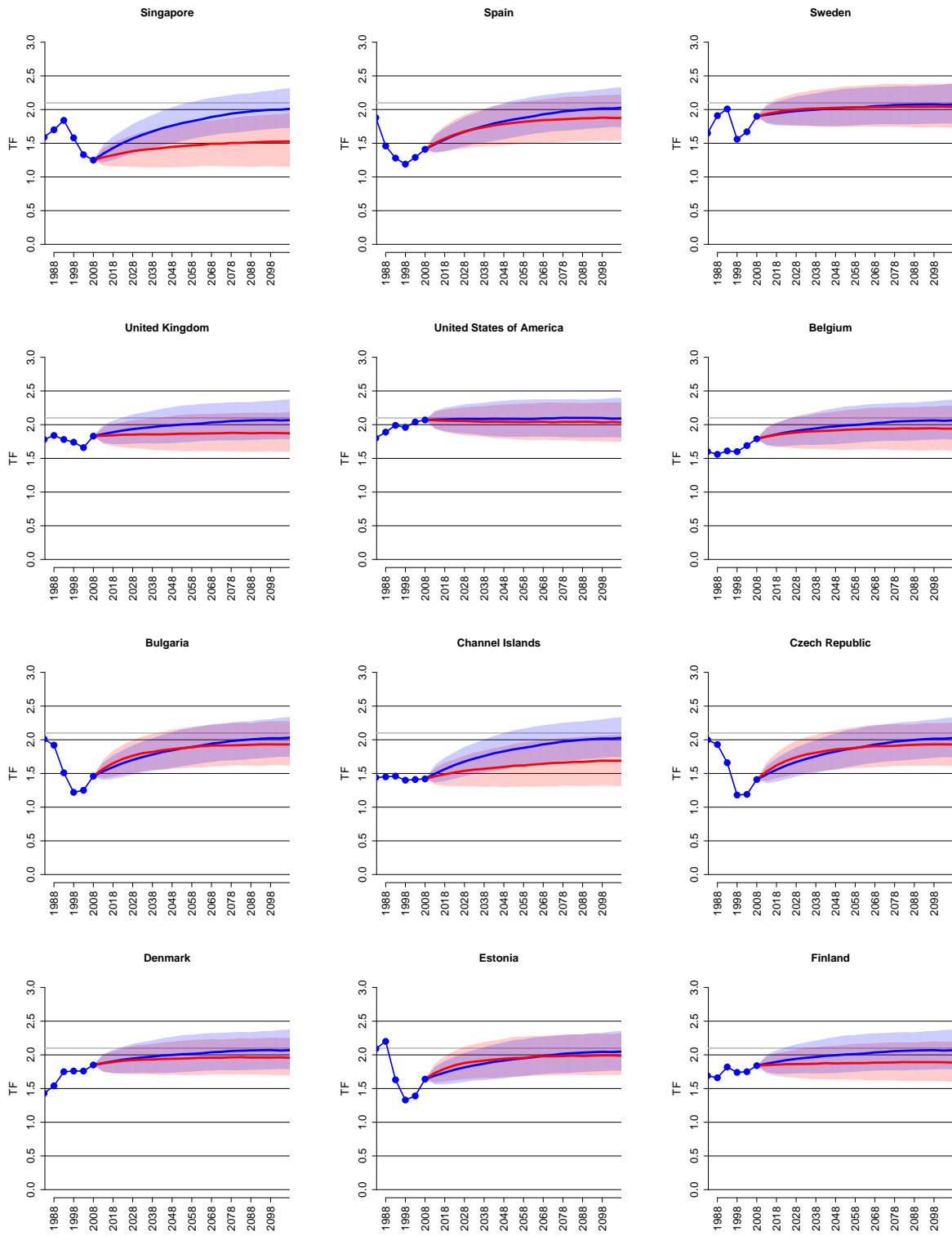


Figure 2, continued on next page

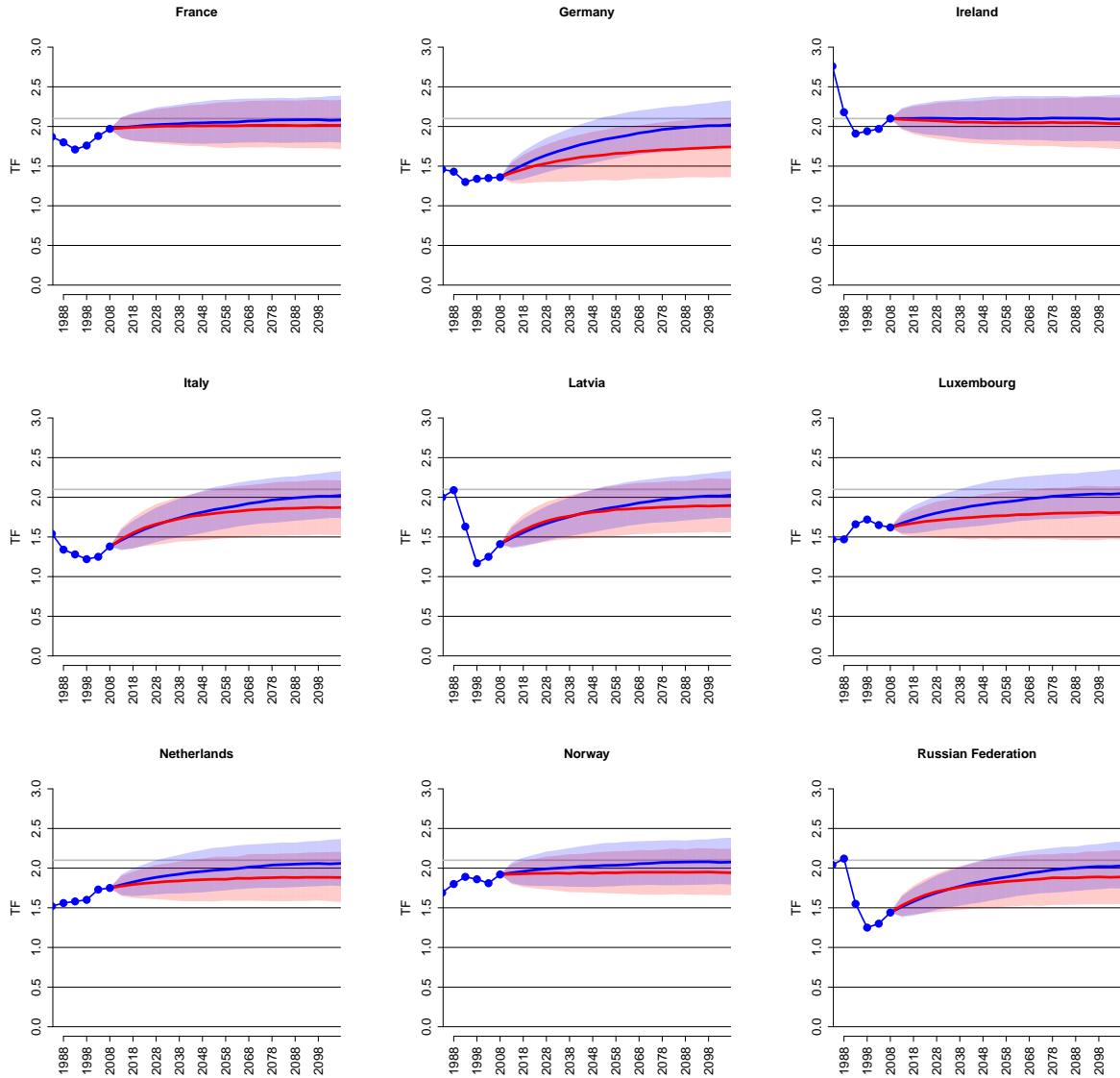


Figure 2: Projections of the TF with 80% projection intervals for all countries that are currently in Phase III under the WPP 2010 model (blue) and the Bayesian hierarchical model (red).

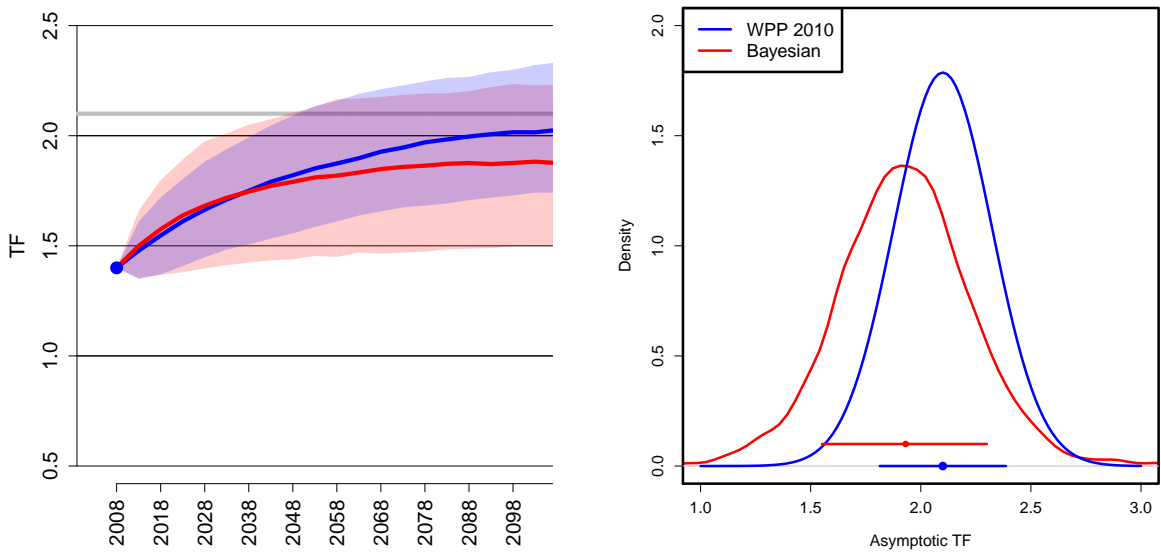


Figure 3: Left: Projections starting at TF of 1.4 in year 2008 (period 2005-2010) by the WPP 2010 method (blue) and the Bayesian hierarchical method (red). Right: Asymptotic distribution for the TF based on the WPP 2010 method (blue) and the Bayesian hierarchical method (red), with 80% PIs (horizontal lines).

4 Discussion

We discussed the current UN projection method for fertility in countries that have completed the fertility transition and the controversy about the ultimate fertility level. We proposed an alternative Bayesian hierarchical projection model, whereby the ultimate fertility levels and autoregressive parameters are country-specific, to allow for variation across countries. Expert opinion is incorporated into the model by setting the upper bound on the ultimate fertility level to 2.1. We found small differences for most post-transition countries (ultimate levels less than 0.25 child lower) but there are exceptions. Most notably, the hierarchical model results in lower fertility projections for Singapore, suggesting that the data provide some support for the contention by Basten et al (2012) and also that the Bayesian hierarchical model can accommodate differences in ultimate fertility levels between countries. For countries that are projected to enter the post-transition phase in the future, we find that the long-term projected median TF for the hierarchical model is 1.9 (1.6, 2.3), compared to 2.1 (1.8, 2.4) for the WPP 2010 method.

Data are key to understanding post-transition fertility dynamics. Unfortunately, our analysis is limited by the sparseness of the data; only 54 changes are observed in 21 countries in Phase III. We plan to investigate the use of sub-regional time series to extend our analysis. In particular, we will supplement our analysis with TF series in urban areas where the post-transition fertility phase has started earlier than at the national level. This extended analysis will shed more light on post-transition fertility dynamics. With more data, we will also be able to validate our projection model by out-of-sample projections, whereby the most recent observation(s) are left out, to verify how well the model projects the left-out observations.

Acknowledgements

This work was supported by a research grant from the National University of Singapore and grants R01 HD054511 and R01 HD070936 from the Eunice Kennedy Shriver National Institute of Child Health and Human Development. Its contents are solely the responsibility of the authors and do not necessarily represent the official views of the Eunice Kennedy Shriver National Institute of Child Health and Human Development. Also, the views expressed in this article are those of the authors and do not necessarily reflect the views of the United Nations. Its contents have not been formally edited and cleared by the United Nations. The designations employed and the presentation of material in this article do not imply the expression of any opinion whatsoever on the part of the United Nations concerning the legal status of any country, territory, city or area or of its authorities, or concerning the delimitation of its frontiers or boundaries. The authors are grateful to Stuart Basten, John Bongaarts, Thomas Buettner, Samuel Clark, Joel Cohen, Gerhard Heilig, Ronald Lee, Nan Li, Hana Ševčíková and Hania Zlotnik for helpful comments and discussions.

References

- Alkema, L., A. E. Raftery, P. Gerland, S. J. Clark, F. Pelletier, T. Buettner, and G. K. Heilig (2011). Probabilistic projections of the total fertility rate for all countries. *De-*

mography 48, 815–839.

- Basten, S., D. Coleman, and B. Gu (2012). Re-examining the fertility assumptions in the UN's 2010 World Population Prospects: Intentions and fertility recovery in East Asia? In *Presented at the Annual Meeting of the Population Association of America*, San Francisco. <http://paa2012.princeton.edu/sessionViewer.aspx?SessionId=112>.
- Lutz, W. (2008). Has korea's fertility reached the bottom? the hypothesis of a 'low fertility trap' in parts of europe and east asia. *Asian Population Studies* 4, 1–4.
- Myrskylä, M., H. P. Kohler, and F. C. Billari (2009). Advances in development reverse fertility declines. *Nature* 460, 741–743.
- United Nations (2011). *World Population Prospects: The 2010 Revision*. New York, NY: United Nations.