Population Momentum with Migration

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Abstract

Population momentum provides an important tool for policy discussions, yet often ignores a critical ingredient to population dynamics: migration. We develop a framework for momentum that incorporates in- and out-migration rates, which allows for a spectrum of momentum values as stationarity is achieved through a combination of fertility and migration rate changes. We apply the framework to Eurostat data, and find that closed-population measures typically underestimate migration-inclusive momentum in these countries. The framework highlights the relationship between momentum and population aging; the policy choice is between high-momentum and older, or low-momentum and younger populations.

1 Introduction

The concept of population momentum refers to the subsequent impacts on population size and age structure of changes in vital rates. Conceived in the context of growing countries, momentum answered the question, "What would happen to population size and age composition if maternity rates immediately fell to replacement level?" Typically, the youthful age distribution of a growing population meant that even if fertility rates moved instantaneously to replacement, the population would continue to grow for a while as the relatively larger young cohorts replaced smaller cohorts at older ages (Preston, 1986). This innovation provided a way to take into account long-run implications of the current age structure and alternative population policies.

Discussions of population momentum usually assume a population closed to migration (Keyfitz, 1971; Bourgeois-Pichat, 1971; Schoen & Kim, 1991; Preston & Guillot, 1997; Espenshade, Olgiati, & Levin, 2011; Blue & Espenshade, 2011). Many countries, however, have nontrivial levels of both in- and out-migration, and simply reducing fertility to replacement as conventionally defined will not

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necessarily lead to zero population growth and a stationary population in the long run. Work has been done to incorporate momentum into a two-region model, with migration between regions (Rogers & Willekens, 1978; Schoen, 2002), but a truly open framework, with the policy perspective from a single state, still does not exist. Here, we develop a measure of momentum that accounts for current migration patterns and explore the implications of these patterns for long-run population size and age distribution. Our framework facilitates a two-pronged approach to population policy, adjusting maternity and/or migration rates to achieve replacement.

2 Expanded Framework

Consider a female population of size P with proportionate age structure $c(x)$, mortality rates $\mu(x)$, in-migration rates $\iota(x)$, out-migration rates $\nu(x)$, and maternity rates $m(x)$, all referring to exact age x. These mortality and migration rates give a migration-inclusive survival function of

$$\pi(a) = e^{-\int_0^a (\mu(x)+\nu(x)-\iota(x))dx}, \tag{1}$$

where $\pi(a)$ is not the mortality-based survival function, $p(a)$, of the population, but rather the migration-inclusive fraction of an original birth cohort that is alive and in the country at age a. Notice that $\pi(a)$ is not necessarily non-increasing. It can rise in any age interval for which the in-migration rate is larger than the sum of mortality and out-migration rates, and $\pi(a)$ can be larger than 1. Still, for a maximum age ω, the migration-inclusive survival function must decline to $\pi(\omega) = 0$. We assume that migrants (both incoming and outgoing) share the average mortality and fertility of the entire population.[1]

This population has an intrinsic growth rate r^* that satisfies (by definition) the equation

$$\int_0^\beta e^{-r^* a}\pi(a)m(a)da = 1, \tag{2}$$

where β is the maximum age of childbearing. We use the notation r^* to indicate that this is the intrinsic growth rate of an *open* population, and not the traditional r of a closed population. Deriving population momentum requires finding changes to the migration-inclusive survival and/or maternity functions that can achieve a new intrinsic growth rate of $r^* = 0$. To begin, we consider two limiting cases.

First, replace the original maternity function with the new rates

$$m_0(a) = e^{-r^* a}m(a).$$

The subscript 0 is used to indicate replacement values in the stationary population, where $r^* = 0$ by definition. With a positive r^*, this corresponds to a percentage reduction in fertility that is proportional to age, with fertility falling

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[1] In the Discussion section we consider an appropriate denominator for immigrant flows.

more sharply at older ages. By construction, $\int_0^\beta \pi(a)m_0(a)da = 1$ (which follows from Equation (2)), and the new intrinsic growth rate of the population is now $r^* = 0$. Reducing fertility rates in this way to achieve replacement, in contrast to the more conventional approach of normalizing the fertility schedule by the net reproduction rate (NRR), was proposed by Mitra (1976) and is more consistent with observed changes in maternity rates during the fertility transition (Coale & Watkins, 1986).

Alternatively, replace the initial migration-inclusive survival function with a new function

$$\begin{aligned}\pi_0(a) &= e^{-r^*a}\pi(a)\\ &= e^{-\int_0^a(\mu(x)+\nu(x)-[\iota(x)-r^*])dx}.\end{aligned}$$

The new intrinsic growth rate of this population is again 0, as $\int_0^\beta \pi_0(a)m(a)da = 1$. If we assume that in-migration is a more important policy variable than out-migration or mortality, then for positive values of r^*, $\pi_0(a)$ corresponds to a reduction of in-migration rates by a constant amount r^* at all ages. We prefer to interpret $\pi_0(a)$ this way rather than as an increase in mortality and/or out-migration.

Now generalize these two polar cases to account for a shifting balance between changing maternity and in-migration rates. Define new maternity and survival functions as

$$m_0(a|\gamma) = e^{-(1-\gamma)r^*a}m(a) \tag{3}$$

$$\pi_0(a|\gamma) = e^{-\gamma r^*a}\pi(a), \tag{4}$$

with the proportion of the rate change absorbed by the migration-inclusive survival rates parameterized as γ. With this expanded framework, there are infinitely many ways of achieving replacement conditions depending on the value of γ and, therefore, on relative changes in fertility versus in-migration. In the extremes, when $\gamma = 0$, all of the work in reaching replacement is being done on the fertility side. If $\gamma = 1$, all of it is being done on the side of in-migration.[2]

When used together, these new maternity and survival rates have an intrinsic growth rate of zero:

$$\begin{aligned}\int_0^\beta \pi_0(a|\gamma)m_0(a|\gamma)da &= \int_0^\beta \left(e^{-\gamma r^*a}\pi(a)\right)\left(e^{-(1-\gamma)r^*a}m(a)\right)da\\ &= \int_0^\beta e^{-(\gamma+(1-\gamma))r^*a}\pi(a)m(a)da\\ &= \int_0^\beta e^{-r^*a}\pi(a)m(a)da\\ &= 1,\end{aligned}$$

where the last step comes from the definition of r^* in Equation (2).

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[2] Note that γ does not have to be bounded by 0 and 1, but could be any real number. Values of γ outside of this interval would correspond to overshooting the adjustment of one function and making up for it with the other. However, it is useful to think of γ as being contained in the interval $[0, 1]$.

2.1 Stationary Age Distributions

In a stationary population with zero intrinsic growth, the age distribution, $c_0(a)$, is proportional to the survival function. In a migration-inclusive stationary population, the age distribution is

$$\begin{aligned} c_0(a|\gamma) &= b_0(\gamma)\pi_0(a|\gamma) \\ &= b_0(\gamma)e^{-\gamma r^* a}\pi(a), \end{aligned} \tag{5}$$

where $b_0(\gamma)$ is the birth rate in the migration-inclusive stationary population and equal to $1/\int_0^\omega \pi_0(a|\gamma)da$.

If $\gamma = 0$, the stationary age distribution is proportional to $\pi_0(a|\gamma = 0) = \pi(a)$, or the original migration-inclusive survival function. If $\gamma = 1$, the stationary age distribution is proportional to $e^{-r^* a}\pi(a)$, which means that the migration-inclusive stationary population and the stable population associated with the initial vital rates $m(a)$ and $\pi(a)$ have the *same* age distribution. This somewhat counterintuitive result when $\gamma = 1$ is explained by the fact that raising or lowering in-migration rates by a constant amount r^* at all ages is equivalent to a "neutral" change in mortality that has no effect on age composition (Coale 1972: 33-34; Tognetti, 1976).

The stationary age distribution, $c_0(a|\gamma)$, becomes younger as γ varies from 0 to 1, as long as $r^* > 0$. It becomes older as γ increases whenever $r^* < 0$. And $c_0(a|\gamma)$ is invariant with respect to changes in γ when $r^* = 0$.

2.2 Migration-Inclusive Momentum

Building on Preston & Guillot (1997) and on Espenshade, Olgiati, & Levin (2011), total momentum of a population with size P and age structure $c(x)$ is, in the migration-inclusive case,

$$M(\gamma) = \frac{S(\gamma)}{P} = \int_0^\beta \frac{c(x)}{c_0(x|\gamma)} \int_x^\beta \pi_0(a|\gamma)m_0(a|\gamma)da\, dx / A_0(\gamma). \tag{6}$$

$S(\gamma)$ is the size of an eventual stationary population that results after converting immediately fertility and in-migration to γ-dependent replacement conditions and then holding fertility, mortality, and both in- and out-migration rates constant until an equilibrium stationary population is achieved. $A_0(\gamma)$ is the mean age of childbearing in the migration-inclusive stationary population and equal to $\int_0^\beta a\pi_0(a|\gamma)m_0(a|\gamma)da$. It is straightforward to show that $A_0(\gamma) = A_0$, or that A_0 does not depend on γ.

Simplifying Equation (6), we have

$$M(\gamma) = \int_0^\beta \frac{c(x)}{c_0(x|\gamma)} \int_x^\beta e^{-r^* a}\pi(a)m(a)da\, dx / A_0, \tag{7}$$

which shows that γ influences total momentum only through the shape of the migration-inclusive stationary population age distribution. Typically, in discussions involving closed populations, an observed population has one and only one momentum value associated with it, because fertility rates are adjusted to replacement by dividing the maternity schedule by the NRR. Here, in the way we are treating open populations, there are many paths to replacement conditions, depending on the value of γ, and each path usually implies a different coefficient of momentum.

Equation (7) shows that total migration-inclusive momentum is a weighted average of age-specific relative deviations between the observed age distribution, $c(x)$, and the migration-inclusive stationary population age distribution, $c_0(x|\gamma)$, where the weights are $w(x) = \int_x^\beta e^{-r^* a}\pi(a)m(a)da/A_0$. These weights sum to unity, they are largest and constant at $1/A_0$ prior to the onset of childbearing, and they decline monotonically to zero as x increases from the beginning (α) to the end (β) of the childbearing years. Therefore, deviations between $c(x)$ and $c_0(x|\gamma)$ that matter most in determining population momentum are those in the first 15 years or so of life. Deviations toward the end of the childbearing years, regardless of their size, contribute comparatively little.

We may ask how total momentum changes with changes in γ. The next section addresses this question more formally. Here we discuss only the direction of the effect. The answer depends on the value of r^*. Suppose $r^* > 0$, so that $c_0(x|\gamma)$ becomes younger as γ increases from 0 to 1. Then, from Equation (7), total momentum falls. Conversely, when $r^* < 0$, an increase in γ corresponds to smaller proportions of infants and children in the migration-inclusive stationary population, so that momentum increases as γ rises. And if $r^* = 0$, there is no change in $c_0(x|\gamma)$ as γ increases, implying that momentum is constant.

Are there observed populations that contain no momentum, that is, in which the coefficient of momentum equals 1.0? One obvious example is if the observed population is already stationary, so that $r^* = 0$ with $\pi(a)$ and $m(a)$. Beyond that, consider an arbitrary initial population that is stable with growth rate r^*. If a migration-inclusive stationary population is achieved by setting $\gamma = 1$, then the stable and stationary populations have identical age distributions and, consequently, zero momentum. In other words, if we imagine the successive momentum values that an initially stable population acquires as γ ranges from 0 to 1, that curve will always be anchored at 1.0 when $\gamma = 1$.

Finally, we noted earlier that $A_0(\gamma) = A_0$, or that the mean age of childbearing in the migration-inclusive stationary population does not depend on γ. Suppose $r^* > 0$ and consider what happens as γ ranges from 1 to 0. The fertility schedule goes from doing none of the work to reach replacement (when $\gamma = 1$) to doing all of it (when $\gamma = 0$). In the process, fertility declines to $e^{-r^* a}m(a)$, and the mean age of the fertility schedule also falls. Simultaneously, the migration-inclusive stationary population grows older. As γ moves toward 0, the stationary population contains relatively more older women in the childbearing ages who are bearing increasingly fewer children compared to younger women, and relatively fewer younger women who are bearing children at increasingly higher rates compared to older women. The falling mean age of the fertility schedule is just sufficient to offset the rising mean age of the migration-inclusive stationary population, leaving the mean age of childbearing unchanged.

More precisely, for a small reduction $\Delta\gamma > 0$ in γ, the maternity rate at age a, $m_0(a|\gamma)$, is lowered when multiplied by the proportion $e^{-\Delta\gamma\, r^* a}$. But the migration-inclusive survivorship to age a, $\pi_0(a|\gamma)$, is raised by the proportion $e^{\Delta\gamma\, r^* a}$, resulting in no change to the product $\pi_0(a|\gamma)m_0(a|\gamma)$ and therefore to A_0. In other words, changes in γ do not alter the age pattern of total births and therefore do not affect the mean age of childbearing. It is easy to see how a similar argument applies, with the direction of effects reversed, if $r^* < 0$. And if $r^* = 0$, neither the fertility schedule nor the stationary population changes as γ varies.

3 Dependence on γ

A novel aspect of our approach to migration-inclusive population momentum is that a given population is usually consistent with many different long-run stationary population sizes and age distributions, depending on how the path to replacement conditions is distributed between adjustments to maternity and survival functions. As γ proceeds from 0 to 1, more of the adjustment is handled by migration rates. Therefore, it is of interest to formalize some of our earlier findings by examining systematically the dependence of stationary age distributions and total momentum values on γ.

3.1 Stationary Age Distributions

Recall that the stationary age distribution is $c_0(x|\gamma) = b_0(\gamma)\pi_0(x|\gamma) = b_0(\gamma)e^{-\gamma r^* x}\pi(x)$, where the birth rate $b_0(\gamma) = 1/\int_0^\omega \pi_0(x|\gamma)dx$. Then the change in the stationary birth rate with respect to a small change in γ is

$$\begin{aligned}\frac{d}{d\gamma}b_0(\gamma) &= r^* b_0(\gamma)^2 \int_0^\omega x e^{-\gamma r^* x}\pi(x)dx \\ &= r^* b_0(\gamma)\int_0^\omega x c_0(x|\gamma)dx.\end{aligned}$$

Notice that the integral is simply the mean age of the migration-inclusive stationary population associated with γ, which we denote as $\bar{x}_0(\gamma)$. Dividing both sides by $b_0(\gamma)$ gives the proportionate change in the birth rate as

$$\frac{d\ \ln b_0(\gamma)}{d\gamma} = r^*\bar{x}_0(\gamma). \tag{8}$$

The proportional change in the birth rate with respect to γ is simply the intrinsic growth rate times the mean age of the stationary population associated with γ. Because the mean age is always positive, the dependence of $b_0(\gamma)$ on γ will have the same sign as r^*. When $r^* > 0$ an increase in γ raises $b_0(\gamma)$, and when $r^* < 0$ an increase in γ lowers $b_0(\gamma)$. And if $r^* = 0$, there is no change in $b_0(\gamma)$

as γ increases. All of this is consistent with our earlier observations at the end of Section 2.1.

More generally, the change in the stationary age structure, or the proportion of the population at age x, with respect to γ is

$$\begin{aligned}\frac{d}{d\gamma}c_0(x|\gamma) &= \frac{d}{d\gamma}\left(e^{-\gamma r^* x}\pi(x)b_0(\gamma)\right)\\ &= -r^* x e^{-\gamma r^* x}\pi(x)b_0(\gamma) + e^{-\gamma r^* x}\pi(x)b_0(\gamma)r^*\bar{x}_0(\gamma)\\ &= -r^* c_0(x|\gamma)\left(x - \bar{x}_0(\gamma)\right).\end{aligned}$$

We can rewrite this derivative as

$$\frac{d\ \ln c_0(x|\gamma)}{d\gamma} = r^*(\bar{x}_0(\gamma) - x). \tag{9}$$

This dependence on γ has the same sign as r^* for ages x younger than the mean age, $\bar{x}_0(\gamma)$, and opposite sign for x older than the mean age. That is, when $r^* > 0$, for ages younger than the stationary mean age, the share of the population in the stationary age distribution grows with respect to γ; for ages older than the stationary mean age, the share shrinks. In other words, the stationary age distribution pivots around its mean age for small changes in γ. Taking a second to step back, this fact is intuitive: in a growing stable population, achieving stationarity means dropping maternity rates and/or net migration rates. Increasing γ means lowering net migration rates more and reducing maternity rates less, which will lead to a younger long-run population. Finally, note that the value of the stationary age distribution when $x = 0$ equals the stationary birth rate. And, when $x = 0$, Equations (8) and (9) become identical.

3.2 Total Momentum

To begin, rewrite the formula for total momentum in Equation (7) as

$$M(\gamma) = \int_0^\beta \frac{c(x)}{c_0(x|\gamma)}\, w(x)\, dx,$$

where $w(x)$ represents a set of age-specific weights,

$$w(x) = \int_x^\beta e^{-r^* a}\pi(a)m(a)da/A_0.$$

Then

$$\frac{d}{d\gamma}M(\gamma) = \int_0^\beta \frac{d}{d\gamma}\left(\frac{c(x)}{c_0(x|\gamma)}\right) w(x)\, dx. \tag{10}$$

The derivative in the integral is

$$\begin{aligned}\frac{d}{d\gamma}\left(\frac{c(x)}{c_0(x|\gamma)}\right) &= \frac{-c(x)}{c_0(x|\gamma)^2}\frac{d}{d\gamma}c_0(x|\gamma)\\ &= \frac{-c(x)}{c_0(x|\gamma)^2}r^*c_0(x|\gamma)\left(\bar{x}_0(\gamma)-x\right)\\ &= \frac{-r^*c(x)}{c_0(x|\gamma)}\left(\bar{x}_0(\gamma)-x\right).\end{aligned}$$

Substituting into Equation (10) gives

$$\frac{d}{d\gamma}M(\gamma) = -r^*\int_0^\beta \frac{c(x)}{c_0(x|\gamma)}\,w(x)\left(\bar{x}_0(\gamma)-x\right)dx. \tag{11}$$

The integral in Equation (11) involves three terms. The first, a ratio of proportionate age distributions, is always positive. The second is a positive weighting function where the weights sum to unity on $0 \le x \le \beta$. The values of $w(x)$ are largest before the onset of childbearing and then decline rapidly toward zero for ages beyond the peak childbearing years. The final term, $(\bar{x}_0(\gamma)-x)$, is positive on $0 \le x < \bar{x}_0(\gamma)$, declines linearly from $\bar{x}_0(\gamma)$ when $x = 0$ to zero when $x = \bar{x}_0(\gamma)$, and turns negative for $x > \bar{x}_0(\gamma)$. Typically $\bar{x}_0(\gamma)$, the mean age of the migration-inclusive stationary population, takes on values that are well into, if not beyond, the childbearing years, and any negative values of $(\bar{x}_0(\gamma)-x)$ receive only a small amount of weight through $w(x)$. In other words, $(\bar{x}_0(\gamma)-x)$ compounds the effect of the weighting function $w(x)$, so that what matters most are values of $c(x)/c_0(x|\gamma)$ early in life. We may conclude that, for a broad range of contemporary human experience, the integral in Equation (11) will be positive.

In short, the change in total momentum with respect to γ will be opposite in sign to r^*. And the absolute value of r^* will amplify or temper the response. Clearly, there is no response if $r^* = 0$.

4 Applications to Eurostat Data

We calculated new migration-inclusive momentum measures and associated stationary age distributions for all European Union countries with complete immigration, emigration, mortality and fertility data, using the Population Database provided by the statistical office of the European Union (Eurostat, 2012). Momentum measures were obtained by projecting populations in discrete five-year intervals into the future until they became stationary, usually after no more than 300 years, and then comparing the long-run population size with the current size.

Table 1 contains summary statistics for the 58 country-year observations in the sample. With the exception of Iceland and Ireland, fertility rates are below replacement (in closed populations), leading to negative values for the intrinsic growth rate r. On the other hand, most of the net migration rates are positive (with the main exceptions of the Baltic states of Estonia, Latvia, and Lithuania), which implies that intrinsic r^* is generally larger than intrinsic r. Luxembourg, Norway, Sweden, and Switzerland exhibit the largest net in-migration rates. Despite the contribution from positive net migration, intrinsic r^* values are typically negative–in 34 out of the 58 observations and in 16 out of 26 countries examined. Even with their recorded excess of immigrants over emigrants, these populations would eventually decline if current rates were held constant. Unlike r or r^*, the observed rate of population growth is usually positive–a phenomenon that is most likely caused by "middle-heavy" age distributions due to fertility declines over the past few decades.

[Table 1 about here]

The last two columns of Table 1 show migration-inclusive momentum values. Whenever $r^* < 0$, momentum is higher for $\gamma = 1$ than for $\gamma = 0$. The relation between momentum values is reversed whenever $r^* > 0$. And when $r^* = 0$ (as it is to three decimals places for Denmark 2009-10 and Finland 2008-10), there is no change in momentum as γ varies between 0 and 1. Moreover, total momentum ranges across the sample from a low of 0.97 to a high of 1.55 when $\gamma = 1$, and between a minimum of 0.57 and a maximum of 1.82 when $\gamma = 0$. Because there are 22 instances in which the value of γ determines whether total momentum is above or below 1.0, for them the choice of γ determines whether the long-run stationary population is larger or smaller than its current size. In the remaining 36 cases, it determines by how much the population grows, that is, how much positive momentum is embedded in the current age structure.

Figure 1 shows in another way the contribution that net migration makes to intrinsic population growth. The figure graphs r^* against r for the points in our sample. In the absence of migration, or if net migration were zero age by age, all points would lie along a 45-degree line. But most of the points in Figure 1 lie above this line, suggesting that net migration is typically positive and adds to intrinsic population growth. There are 23 cases that appear in the upper-left-hand quadrant where $r < 0$ and $r^* > 0$ (including, for example, all three years for Denmark, Finland, Luxembourg, Norway, and Sweden). These are instances where net migration is sufficiently important to convert what would be intrinsic population decline in the absence of migration into intrinsic, long-term population growth.

[Figure 1 about here]

Figure 2a shows for various values of γ the long-run stationary age distributions for Germany, using 2009 demographic rates. Germany has a migration-inclusive intrinsic growth rate of -0.011, meaning that if current demographic rates hold, the long-run population would shrink by 1.1% per year. Stationarity requires raising this intrinsic rate to zero, by increasing maternity rates and/or the net influx of migrants. Notice that $\gamma = 0$, which corresponds to attaining stationarity entirely through increasing maternity rates, leads to a much younger age distribution. Conversely, $\gamma = 1$, corresponding to achieving stationarity entirely through altering migration rates, leads to a substantially older population. These changes are consistent with Equation (9) for negative r^*. This also makes intuitive sense. Raising fertility adds more people at age $x = 0$, whereas in-migrants are distributed all along the age spectrum. The noticeable pivot point in the plot is $\bar{x}_0(\gamma)$, though this is not truly a single point, as the mean age under stationarity will in general change with γ.

[Figure 2 about here]

Calculations of total momentum show how the size of the long-run stationary population of Germany changes as γ varies (see Figure 2c). With $\gamma = 0$, momentum is 0.87 indicating that Germany's ultimate population size will be 13% smaller than its current size. Raising γ, and therefore the dependence on changes in migration rates, causes this momentum to increase. Achieving stationarity entirely through migration leads to a momentum value of 1.25, meaning that the population will grow before settling at a population size 25% larger than the current one.

Similar graphs for Iceland in 2008 are shown in Figures 2b and 2d. The country had a positive intrinsic growth rate of 0.015, meaning that the population, under 2008 demographic rates, would grow 1.5% annually in the long run. The dependence of the age structure on γ is now reversed; generating stationarity entirely by changing maternity rates ($\gamma = 0$) leads to an older population, because in the case of a country with positive r^*, achieving stationarity means lowering maternity rates. Figure 2d shows that total momentum decreases with γ, though it remains greater than 1.0, indicating that the population will reach stationarity at a population size larger than the current one.

Equation (11) showed that the sign of the change in momentum with respect to γ is opposite in sign to the intrinsic growth rate r^*. Germany 2009 and Iceland 2008 exhibit this pattern. Figure 3 illustrates that this is universally true in our sample; all countries with negative intrinsic growth rates have gains in momentum when $\gamma = 1$, and countries with positive intrinsic growth rates experience declines. It is also clear in Figure 3 that the absolute value of r^* influences the size of the gain or loss.

[Figure 3 about here]

5 Policy Implications

Most policy discussions of population momentum assume a world in which there is no international migration (Bongaarts & Bulatao, 1999; Bongaarts, 2007, 2009), though the United Nations (2001) work on "replacement migration" could be considered an exception (Espenshade, 2001). Fertility is typically reduced (or sometimes increased) instantaneously by dividing fertility rates by the net reproduction rate, and then mortality and the new fertility rates are held constant until a stationary population is achieved. Comparing the size of the ultimate stationary population with the size of the initial population yields the single measure of population momentum as conventionally defined.

We have shown that the range of policy options is greatly expanded when closed populations are opened up to include international migration. Now there are multiple paths to reaching replacement conditions, depending on how that work is divided between fertility changes on the one hand and changes in international migration on the other. The measure we assign to total population momentum typically responds to this balance, as does the shape of the long-run stationary age distribution. Especially in circumstances in which fertility is resistant to change, policymakers have additional flexibility in reaching replacement through alterations in international migration.

At the same time, it is important to recognize that choices about long-run stationary population size and age distribution cannot be made independently. A particular value for γ determines both of these simultaneously. Moreover, varying γ moves a population along a stationary-population continuum that ranges from smaller/younger to larger/older. Becoming smaller and older is not possible, nor is becoming larger and younger. This is true whether r^* is positive or negative. But the sign of r^* does matter. When $r^* > 0$, increasing γ from 0 to 1 makes a stationary population smaller and younger. But an increase in γ when $r^* < 0$ creates an older and larger stationary population. If, for policy purposes, it is desirable to have a stationary population that is as small and youthful as possible, then this objective is achieved with $\gamma = 1$ when $r^* > 0$. In other words, the move to replacement is achieved entirely by lowering in-migration. But if $r^* < 0$, then a small and young population is reached when $\gamma = 0$, that is, by raising fertility and leaving in-migration rates alone.

These principles are illustrated by our two case studies in Figure 4. Begin with Germany 2009 whose r^*-value is negative (-0.011). The blue lines trace alternative combinations of momentum coefficients and the percentages of the long-run stationary population under age 15 (the solid line) and over age 75 (the dashed line). As γ increases from 0 to 1, total momentum goes up from 0.87 to 1.25, and the population becomes older as evidenced by an increase in the elderly percentage from 13.5 to 20.9, or by a decline in the percentage under 15 from 16.9 to 10.6. The red lines show that relationships with changes in γ are reversed for Iceland 2008 whose r^*-value is positive (0.015). Now, as γ increases from 0 to 1, total momentum falls from 1.66 to 1.08, and the population becomes younger. The elderly share falls from 16.2 to 8.7 percent, while the youthful share rises from 11.7 to 20.2 percent. In neither case does a stationary population become smaller and older or larger and younger as γ increases. Finally, note that the youth and elderly proportions are approximately balanced in Germany when $\gamma = 0.25$, and in Iceland equal shares occur when γ is about 0.28.

[Figure 4 about here]

Figure 5 plots the closed-population momentum value and the range of momentum values in an open population as γ varies between 0 and 1 for each of the 58 country-year observations, arrayed from left to right in ascending order of their migration-inclusive intrinsic growth rates, r^*. As long as $r^* < 0$, open-population momentum is largest when $\gamma = 1$. But when $r^* > 0$, open momentum is maximized for $\gamma = 0$. Several conclusions can be drawn from the data in Figure 5. First, they satisfy a simple consistency check. There are four cases where net migration is approximately zero and intrinsic r and r^* values are roughly the same (Bulgaria 2008, Germany 2008, Macedonia 2008, and Poland 2008). In these instances, momentum in the closed population–obtained by normalizing the fertility schedule by the NRR–should be approximately equal to momentum in the open population when $\gamma = 0$, that is, when fertility is doing all the adjusting to reach replacement. And, indeed, this is the case.

[Figure 5 about here]

Second, the discussion following Equation (11) showed that the change in total momentum with respect to changes in γ is a positive function of $-r^*$. The largest values for $M(\gamma)$ occur when $\gamma = 1$ for $r^* < 0$, and when $\gamma = 0$ for $r^* > 0$. When $r^* = 0$, $M(\gamma)$ is invariant to changes in γ. Moreover, the farther removed r^* is from 0, the more responsive $M(\gamma)$ is to changes in γ. These conclusions, based on analytic work, are completely consistent with patterns observed in Figure 5.

Third, values for total momentum in both open and closed populations are larger when r^* is positive than when it is negative. This is especially true for $M(\gamma = 0)$ as r^* increases beyond 0. Total momentum in closed populations depends on the ratio between the observed age distribution $c(x)$ and the stationary age distribution $c_0(x)$ at younger ages (Preston & Guillot, 1997; Espenshade et al., 2011). Because the level of mortality does not vary much across the sample, momentum in Figure 5 is influenced mainly by the shape of the observed age distribution. Countries that are comparatively young as measured by their mean ages (Ireland, Iceland, and Cyprus) tend to have among the highest closed-population momentum values. And older populations (for example, Bulgaria, Latvia, Hungary, and Germany) have among the lowest momentum values (typically below 1.0). More generally, higher values of r^* are produced by higher values of TFR and net in-migration. These in turn lead to younger age distributions for $c(x)$, which imply higher values for momentum in closed populations.

On the other hand, as we showed in Equation (7), momentum in an open population depends on relative deviations between $c(x)$ and the migration-inclusive stationary age distribution, $c_0(x|\gamma)$, at younger ages. With $\gamma = 0$, fertility is doing all the work in getting to replacement. So $c_0(x|\gamma = 0)$ is based on the observed migration-inclusive survival schedule. As we move to the right in Figure 5 beyond $r^* = 0$, net in-migration takes on larger values, effectively lowering the migration-inclusive force of mortality schedule, extending survivorship, and creating an older $c_0(x|\gamma)$. Therefore, as r^* grows, $c(x)$ becomes younger while $c_0(x|\gamma)$ grows older, leading to larger values for $M(\gamma = 0)$. When $\gamma = 1$, moving to larger values of r^* creates a younger $c_0(x|\gamma)$. The effect on $M(\gamma = 1)$ is likely to be less dramatic now because $c(x)$ and $c_0(x|\gamma)$ are moving in the same direction. The behaviors of $M(\gamma = 0)$ and $M(\gamma = 1)$ in Figure 5 for positive values of r^* are consistent with this analysis.

Fourth, and most important for policy purposes, relying on conventional measures of population momentum can be seriously misleading. In particular, total momentum that incorporates migration is grossly underestimated by analyses that assume a closed population, at least for the Eurostat countries considered here. The value for closed-population momentum is near the midpoint, or comfortably within the range, of high-to-low open-population momentum in just 6 out of 58 cases, or roughly 10 percent of the time (Lithuania 2010, Ireland all years, Cypress 2008, and Iceland 2008). In 18 cases, it is within the range but lies close to the minimum. In nearly 60 percent (34 out of 58 cases) of the country-years for which we have data, the value for closed-population momentum falls below the open-population momentum range, and it does so by a substantial margin in roughly two-thirds of these instances (22 of 34). In brief, we would understate open-population momentum in roughly 90 percent of our observations by relying on a closed-population estimate. Moreover, in those 34 instances in which the closed-population estimate is out of range, in slightly more than half of these cases (18 of 34), and in 31 percent of all cases, both high and low open-population estimates suggest that momentum is positive, whereas a closed-population estimate points in a negative direction. Here, both the magnitude and the direction of the closed-population effect are incorrect.

To make sense of these patterns, consider two cases. First, when $r^* \geq 0$, net in-migration in our Eurostat data is uniformly positive. To a good first approximation, if r^* is less than net migration, then the migration-inclusive stationary population is older than the migration-exclusive stationary population, and closed-population momentum is less than open-population momentum, $M(\gamma = 1)$. Closed-population momentum is roughly equal to $M(\gamma = 1)$ when r^* equals net migration. And when r^* is greater than net in-migration, the migration-inclusive stationary population is younger than the migration-exclusive stationary population, which implies that closed-population momentum will be greater than $M(\gamma = 1)$. To illustrate, Italy in 2008 had $r^* = 0.001$ and a net in-migration rate of 0.008. Its open-population momentum with $\gamma = 1$ was 1.146 compared with a closed-population momentum value of 0.871. By contrast, for Iceland in 2008, $r^* = 0.015$ and net migration = 0.009. Here, the momenta values are reversed with open-population momentum of 1.082 and closed-population momentum of 1.218. Luxembourg is the only country for which these relations do not hold, but they are supported in the remaining 21 out of 24 cases for which $r^* \geq 0$.

The second case involves $r^* < 0$ in Figure 5. Here the minimum value for $M(\gamma)$ occurs when $\gamma = 0$. Again, to a good approximation, if net in-migration is positive, closed-population momentum is less than $M(\gamma = 0)$; if net in-migration is zero, the two momenta values are equal; and if net in-migration is negative, then closed-population momentum will be greater than $M(\gamma = 0)$. As examples, Lithuania in 2010 had a net migration rate of -0.023, and the rate in Ireland in 2010 was -0.006. In both cases, closed-population momentum exceeded $M(\gamma = 0)$, by 0.904 versus 0.571 and 1.201 versus 1.018, respectively. But in Malta 2010, net migration was 0.007, and open-population momentum (1.257) exceeded its closed-population counterpart (1.006). Each of the 34 country-years for which $r^* < 0$ obeys these principles, except for Cypress 2008.

Up to this point, we have implicitly assumed that γ is free to fluctuate between 0 and 1. But there may be conditions that constrain this variation. Suppose, for example, that $r^* > 0$, in which case one way of moving to replacement conditions is to reduce immigration. For the sake of argument, and to facilitate the analysis, we have assumed that when immigration changes, immigration rates move up or down by a constant amount at all ages–in other words, a "neutral" change in immigration that has no effect on population age structure. But if immigration rates under initial conditions are not sufficiently high, then it may not be possible to lower them enough to achieve stationarity by themselves. We can lower the rates only by as much as the smallest value of the immigration rate below the oldest age of childbearing. This creates a constraint of $min_{x\in[0,\beta]}(\iota(x)) - \gamma r^* \geq 0$, which for positive r^* becomes $\gamma \leq min_{x\in[0,\beta]}(\iota(x))/r^*$. In short, depending on individual-country circumstances, it may not be possible to rely exclusively, or even primarily, on lowering immigration to achieve replacement conditions. In these situations, there could be an upper limit on the value of γ that is less than 1.0.

6 Discussion

Most accounts of population momentum assume closed populations and ignore the effects of international migration. Our paper extends previous research in a way that permits both fertility and migration to be modified to achieve replacement conditions. Our results indicate that the size and age structure of the eventual long-run stationary population depend on how the task of reaching replacement is divided between fertility and migration change. In addition, the paper shows that the size of the stationary population cannot be divorced from its age distribution. Altering the balance between fertility and migration change to reach replacement makes stationary populations either smaller and more youthful or larger and older.

Research since the seminal paper by Keyfitz (1971) on momentum in a stable population has branched off in three directions. First, authors have examined alternative paths to replacement fertility in a single-region, closed population. Mitra (1976) explored the effect of an abrupt, but exponential, fertility decline. Others have studied the impact of a gradual transition to replacement (Frauenthal, 1975; Schoen & Kim, 1998; O'Neill, Scherbov, & Lutz, 1999; Li & Tuljapurkar, 1999, 2000; Goldstein, 2002; Goldstein & Stecklov, 2002; Schoen & Jonsson, 2003). It may be useful to consider gradual transitions to replacement in fertility-migration rate combinations.

Second, Rogers (1975) provided a framework for thinking about momentum in a multiregional setting by describing the characteristics of a stationary multiregional population system. Rogers & Willekens (1978) and Rogers (1995) use this multistate framework to investigate population momentum in a two-region (urban-rural) population for India in which fertility adjusts instantaneously and proportionally to replacement. Schoen's (2002, 2006) modification to a two-region model involves abrupt fertility declines that are proportionately larger at older ages, a pattern of change "similar to that in the Coale-Trussell model fertility schedules" (Schoen, 2002: 54).

Third, researchers have discussed how to incorporate flows of international migrants into multiregional models. Raymer, Abel, & Rogers (2012) describe three options: (1) incorporate net international migration rates into the Leslie growth matrix, (2) separate net migration into immigration and emigration rates, and (3) use immigration counts and emigration rates (pp. 2671, 2682). These alternatives raise a question about the appropriate at-risk population. Options 1 and 2 implicitly assume that the host population is the relevant one for both emigrants and immigrants, whereas option 3 is silent on the at-risk population for immigrants (see also Espenshade, Bouvier, & Arthur, 1982; Rogers, 1990). The second alternative seems like a reasonable assumption for emigrants, and it is consistent with multistate models in which there are no in-migration rates, only destination-specific out-migration rates applied to the correct at-risk populations (Raymer et al., 2012: 2670). And it may be appropriate for immigrants as well, especially in cases of countries that are faced with large flows of potential in-migrants and choose to restrict these flows based on, among other things, the size and age structure of their own populations.

Our paper adopts option 2 for reasons given above. But a fourth alternative also suggests itself. To capture flows of international migrants, why not simply expand an N-dimensional multiregional system to $N+1$ dimensions, where the additional region represents "the rest of the world"? In this context in-migrants to the receiving nation are potential out-migrants from the rest of the world. The latter out-migration rates would reflect the propensity to migrate; they would be applied to the correct populations at-risk; and they could be constrained to the extent that receiving nations place restrictions on immigrant flows.

Policymakers are concerned with the efficacy and cost-effectiveness of policies to influence fertility and immigration, as well as with the implications that higher rates of immigration have for receiving nations. An elaboration of population momentum in migration-inclusive situations cannot address these additional concerns, but it can show the long-run implications for population size and structure of whatever rate changes are considered desirable and feasible.

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Table 1: Summary demographic statistics for the sample (N = 58).

Country & Year	Observed Values				Intrinsic Growth Rates[a]		Migration-Inclusive Momentum	
	Mean Age	Period TFR	Net Mig. Rate[b]	Growth Rate[c]	r	r^*	$\gamma = 0$	$\gamma = 1$
Austria 2008	47.0	1.41	0.005	0.006	−0.013	−0.005	1.10	1.28
Austria 2009	47.2	1.39	0.002	0.003	−0.013	−0.008	1.00	1.29
Austria 2010	47.4	1.44	0.003	0.004	−0.012	−0.007	1.02	1.28
Bulgaria 2008	48.0	1.49	−0.000	−0.003	−0.013	−0.013	0.81	1.17
Croatia 2008	46.8	1.47	0.001	0.000	−0.012	−0.009	0.91	1.17
Cyprus 2008	42.9	1.47	0.001	0.007	−0.011	−0.003	1.09	1.17
Cyprus 2009	43.2	1.52	0.002	0.009	−0.010	−0.001	1.16	1.20
Denmark 2008	45.5	1.89	0.003	0.007	−0.003	0.003	1.11	1.02
Denmark 2009	45.6	1.83	0.003	0.006	−0.004	0.000	1.07	1.06
Denmark 2010	45.7	1.86	0.002	0.006	−0.004	0.000	1.06	1.05
Estonia 2008	47.5	1.66	−0.001	−0.000	−0.008	−0.009	0.86	1.11
Estonia 2009	47.7	1.63	−0.001	−0.001	−0.008	−0.010	0.86	1.12
Estonia 2010	47.8	1.64	−0.002	−0.001	−0.008	−0.011	0.84	1.14
Finland 2008	46.9	1.84	0.002	0.006	−0.004	0.000	1.06	1.05
Finland 2009	47.1	1.86	0.003	0.006	−0.004	0.000	1.06	1.04
Finland 2010	47.2	1.87	0.002	0.006	−0.004	0.000	1.05	1.04
Germany 2008	48.9	1.37	−0.000	−0.001	−0.014	−0.013	0.85	1.25
Germany 2009	49.1	1.35	0.001	0.000	−0.014	−0.011	0.87	1.25
Hungary 2008	46.7	1.35	0.002	0.000	−0.015	−0.011	0.91	1.23
Hungary 2009	46.8	1.33	0.001	−0.001	−0.015	−0.012	0.88	1.24
Iceland 2008	41.4	2.16	0.009	0.019	0.001	0.015	1.66	1.08

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Country & Year	Observed Values				Intrinsic Growth Rates[a]		Migration-Inclusive Momentum	
	Mean Age	Period TFR	Net Mig. Rate[b]	Growth Rate[c]	r	r^*	$\gamma = 0$	$\gamma = 1$
Ireland 2008	41.0	2.07	0.004	0.016	−0.000	0.005	1.34	1.17
Ireland 2009	41.1	2.08	−0.003	0.008	−0.000	−0.005	1.11	1.27
Ireland 2010	41.3	2.08	−0.006	0.005	−0.000	−0.012	1.02	1.37
Italy 2008	48.9	1.42	0.008	0.010	−0.012	0.001	1.18	1.15
Latvia 2008	47.8	1.45	−0.002	−0.004	−0.013	−0.014	0.81	1.19
Latvia 2009	48.0	1.32	−0.002	−0.005	−0.016	−0.019	0.81	1.38
Lithuania 2008	46.6	1.47	−0.003	−0.004	−0.012	−0.016	0.84	1.31
Lithuania 2009	46.8	1.55	−0.004	−0.004	−0.010	−0.016	0.81	1.25
Lithuania 2010	47.1	1.55	−0.023	−0.023	−0.010	−0.040	0.57	1.38
Luxembourg 2008	44.6	1.61	0.015	0.020	−0.008	0.016	1.82	1.10
Luxembourg 2009	44.6	1.59	0.013	0.018	−0.009	0.013	1.64	1.09
Luxembourg 2010	44.8	1.63	0.014	0.019	−0.008	0.016	1.73	1.06
Macedonia 2008	42.3	1.47	0.000	0.003	−0.012	−0.012	1.06	1.50
Malta 2008	45.4	1.45	0.006	0.009	−0.012	−0.004	1.22	1.38
Malta 2009	45.7	1.44	−0.001	0.003	−0.013	−0.012	0.99	1.40
Malta 2010	46.0	1.38	0.007	0.011	−0.014	−0.005	1.26	1.50
Netherlands 2008	45.3	1.76	0.003	0.007	−0.005	0.001	1.13	1.09
Netherlands 2009	45.5	1.78	0.003	0.007	−0.005	0.001	1.10	1.09
Norway 2008	44.5	1.95	0.008	0.014	−0.002	0.012	1.47	1.04
Norway 2009	44.5	1.98	0.008	0.014	−0.002	0.011	1.43	1.03
Norway 2010	44.5	1.94	0.008	0.014	−0.002	0.011	1.46	1.05
Poland 2008	45.3	1.40	−0.000	0.002	−0.014	−0.014	0.97	1.49

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Table 1 – continued from previous page

Country & Year	Observed Values				Intrinsic Growth Rates[a]		Migration-Inclusive Momentum	
	Mean Age	Period TFR	Net Mig. Rate[b]	Growth Rate[c]	r	r^*	$\gamma = 0$	$\gamma = 1$
Slovakia 2008	44.4	1.33	0.001	0.003	−0.016	−0.014	1.03	1.55
Slovakia 2009	44.7	1.42	0.001	0.004	−0.013	−0.012	1.02	1.42
Slovakia 2010	44.9	1.41	0.001	0.003	−0.013	−0.012	1.00	1.41
Slovenia 2008	47.5	1.53	0.003	0.006	−0.010	−0.004	0.98	1.08
Slovenia 2009	47.7	1.54	0.003	0.006	−0.010	−0.004	0.99	1.11
Slovenia 2010	47.8	1.58	0.001	0.004	−0.009	−0.006	0.93	1.10
Spain 2008	46.6	1.47	0.010	0.015	−0.011	0.006	1.35	1.12
Spain 2009	46.7	1.40	0.005	0.009	−0.012	−0.004	1.13	1.27
Spain 2010	46.8	1.39	0.003	0.007	−0.013	−0.008	1.05	1.35
Sweden 2008	46.5	1.90	0.006	0.010	−0.003	0.008	1.25	0.99
Sweden 2009	46.5	1.93	0.007	0.011	−0.002	0.010	1.29	0.97
Sweden 2010	46.5	1.98	0.005	0.010	−0.002	0.008	1.22	0.97
Switzerland 2008	46.7	1.48	0.012	0.015	−0.011	0.009	1.52	1.12
Switzerland 2009	46.8	1.49	0.009	0.013	−0.010	0.006	1.38	1.13
United Kingdom 2009	45.5	1.94	0.003	0.008	−0.002	0.004	1.14	1.01

a - r is the intrinsic growth rate in the closed population, ignoring emigration and immigration; r^* is the intrinsic growth rate in the migration-inclusive population.

b - Net migration rate, calculated as annual net international migration divided by mid-year population size. Positive values represent net inflows.

c - The annual population growth rate, including net migration.

Source: Eurostat (2012) and authors' calculations.

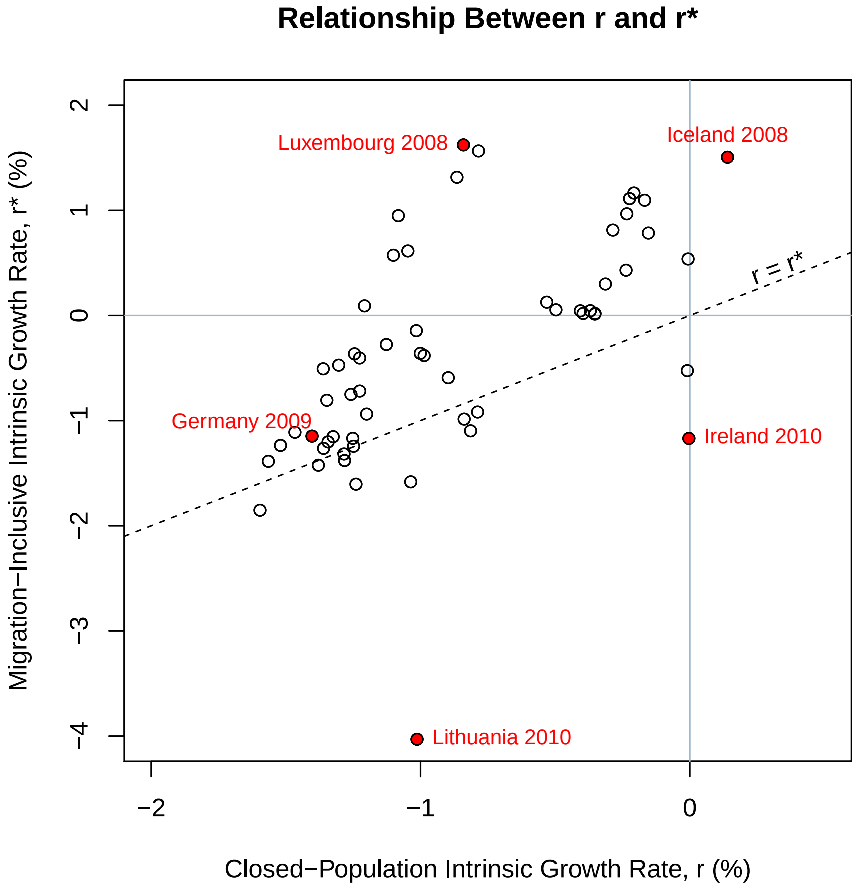
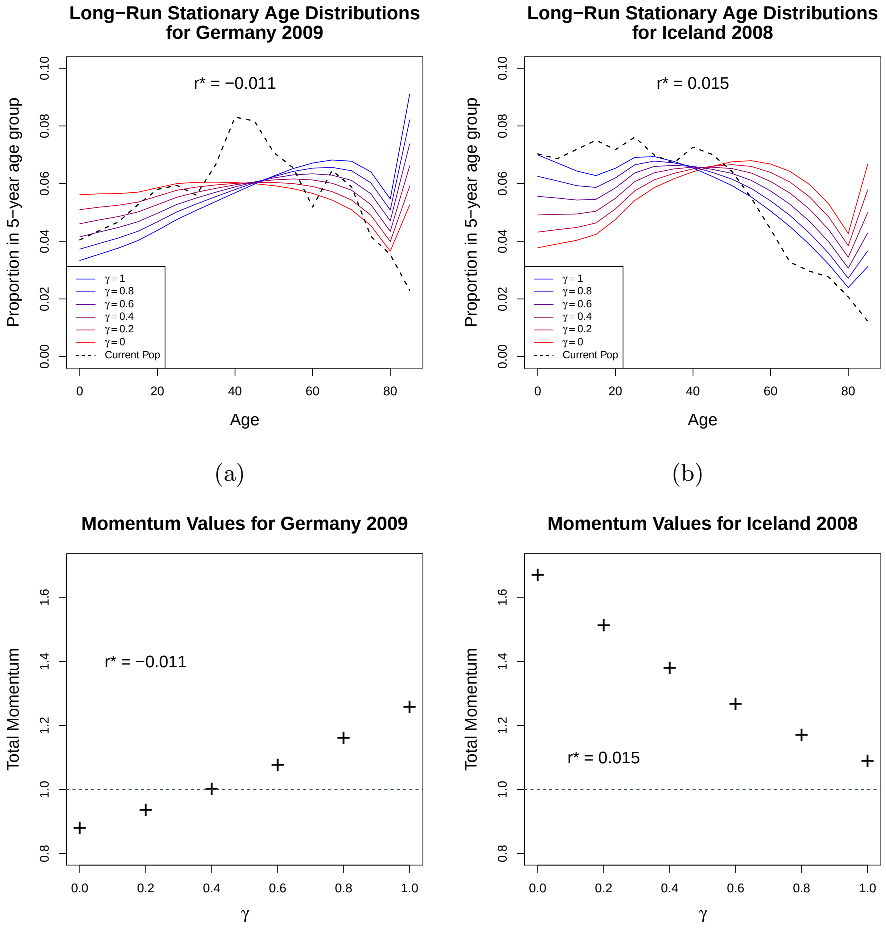


Figure 1: Intrinsic growth rates with migration (r^*) and without migration (r), N = 58. Two case studies (Germany 2009, Iceland 2008) and selected outliers are identified.



(a) (b)

(c) (d)

Figure 2: Long-run stationary population age distributions and total momentum values for Germany 2009 and Iceland 2008.

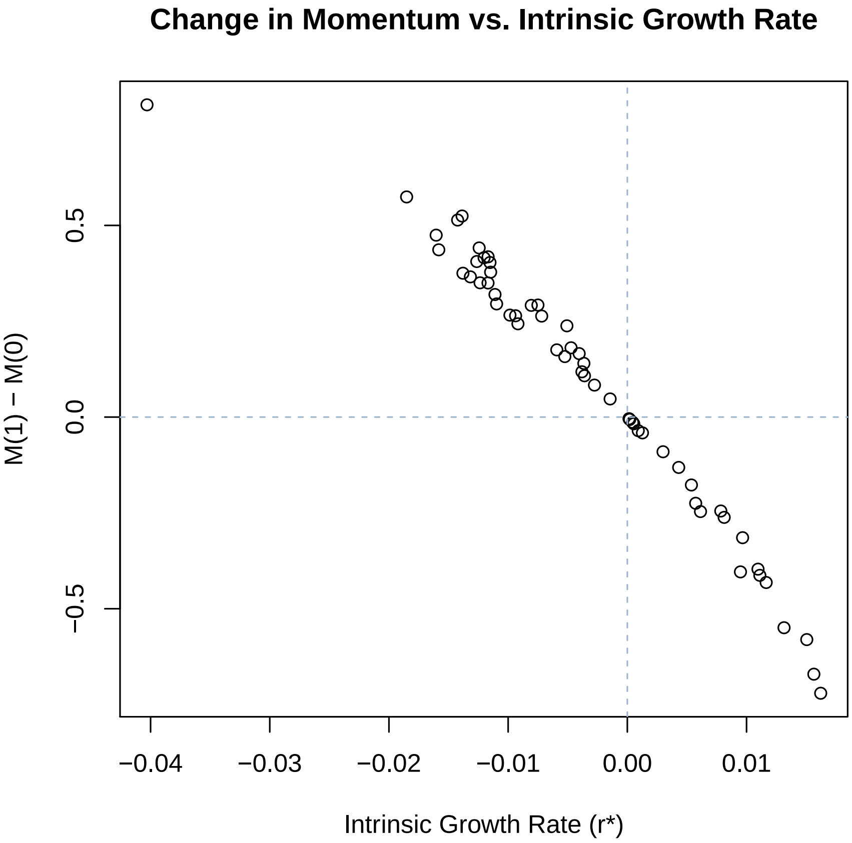


Figure 3: Plot of the difference between momentum for $\gamma = 1$ and $\gamma = 0$, as a function of the intrinsic growth rate r^*, for European Union country-years in the sample (N = 58).

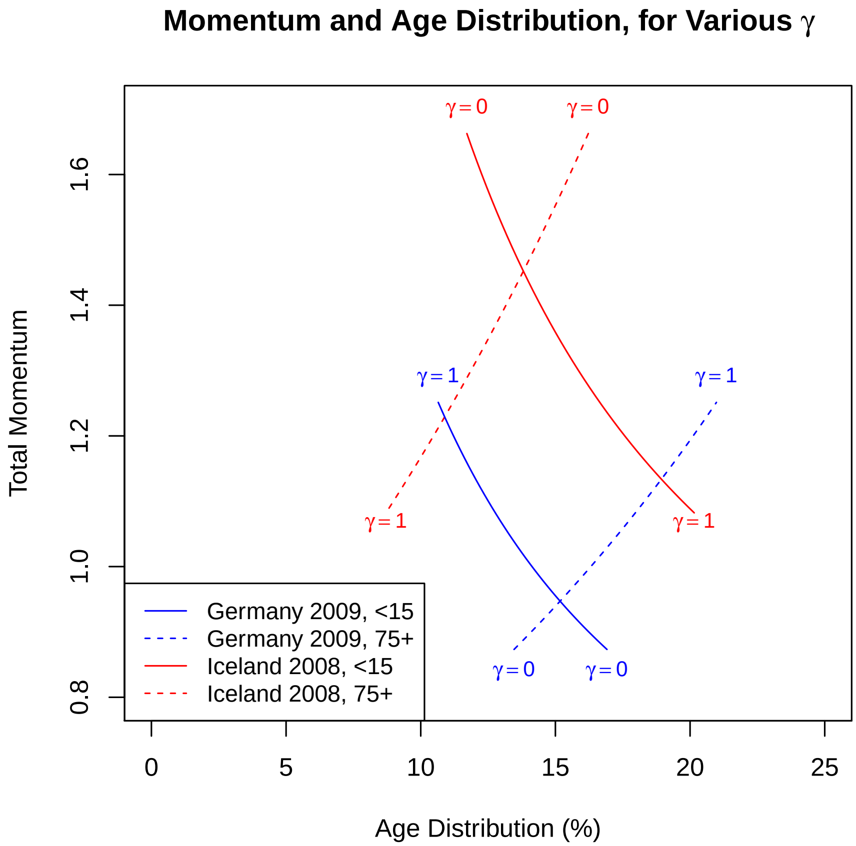


Figure 4: Total momentum values and youthfulness of long-run stationary populations as γ varies between 0 and 1, Germany 2009 and Iceland 2008.

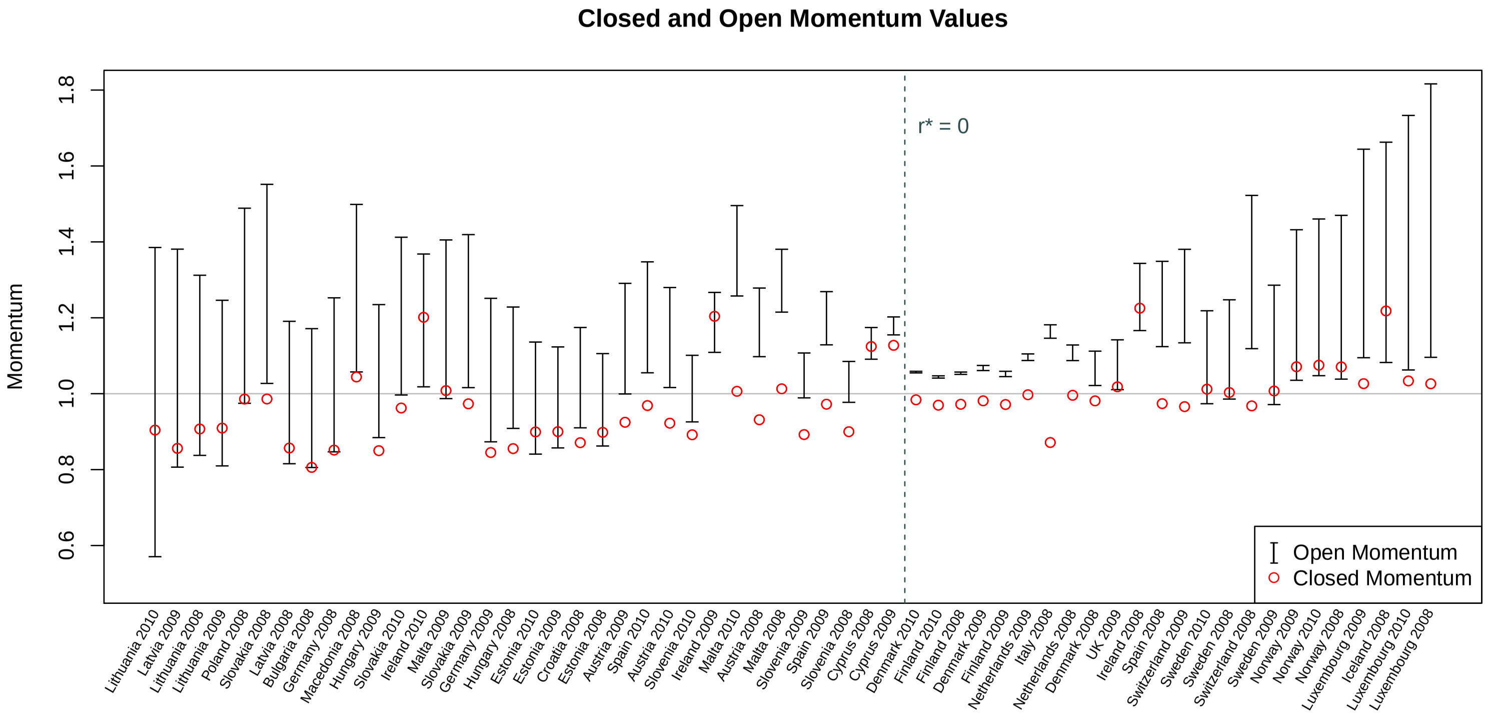


Figure 5: Range of momentum values obtained by varying γ between 0 and 1 in migration-inclusive populations compared to total momentum calculated for closed populations. Country-years are arrayed from left to right in ascending values for r^*.