A mortality model with delayed deaths and cure (Extended Abstract)

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Abstract

We model individual lifetimes by partially adopting the Strehler-Mildvan framework of shocks (Strehler and Mildvan 1960), which lead to death when they exceed one's vitality at the moment of occurrence. The conceptual difference that we offer is that i) shocks are treated as events that initiate deterioration processes which affect one's vitality with a delay; ii) a organism can fully recover with a certain probability after a shock occurs. Our goals are: i) to construct an adequate probabilistic model with delayed deaths and cure (DDC) and study the shape of the population's hazard for different delay distributions (with and without cure), ii) to characterize the class of DDC models which correspond to the frailty model by Vaupel et al. (1979), and iii) to simulate cohorts according to DDC and analyze the mortality patterns for the aggregated data by periods.

Let initiating events (shocks) are modelled by a non-homogeneous process $N(t)$, $t \geq 0$ with a Gompertz rate $\mu(t) = ae^{bt}$. Without loss of generality, we can order the corresponding event times by $T_1 < T_2 < \ldots$ When a shock T_i occurs, it triggers a deterioration process, which leads to death after $D(T_i)$ time units, where $D(T_i)$ is a non-negative random variable with a p.d.f. $g(x)$. Assume the organism can completely recover from the shock with a probability $p(x)$, where x measures time *after* the shock occurs. If D_C denotes the time from the occurrence of the shock to the eventual death caused by this particular shock, then its survival function $\overline{G}_C(x)$ can be expressed as

$$\overline{G}_C(x) = 1 - \int_0^x p(u)g(u)du.$$

Note that D_C is an improper random variable because $P(D_C \equiv \infty) > 0$ as a result of the curing mechanism. Under certain independence assumptions Cha and Finkelstein (2012) proved that the lifetime of the organism can be described by a survival function

$$P(T \geq t) = \exp\left\{ - \int_0^t \overline{G}_C(t-u)\mu(u)du \right\}$$

and a hazard rate (force of mortality)

$$\lambda(t) = \int_0^t p(t-u)g(t-u)\mu(u)du.$$

Suppose there is no curing mechanism, i.e. $p(x) = 1$. Then if $D(T_i) \sim \text{Exp}(\lambda)$

$$\lambda_{\text{Exp}}(t) = \frac{\lambda a}{b+\lambda}\left(e^{bt} - e^{-\lambda t}\right)$$

and if $D(T_i) \sim \Gamma(k, \lambda)$

$$\lambda_{\Gamma}(t) = \frac{ae^{bt}}{\Gamma(k)} \left(1 + \frac{b}{\lambda}\right)^{-k} \gamma(k, (\lambda+b)t)\,,$$

where $\gamma(\cdot,\cdot)$ is the lower incomplete gamma function.
The hazard rate of the DDC model with exponentially distributed delays and no cure $\lambda_{\text{Exp}}(t)$ differs from the Gompertz $\mu(t) = ae^{bt}$ by a factor of $\lambda\left(1 - e^{-(b+\lambda)t}\right)/(b+\lambda)$. This does not change, though, the exponentiallly increasing behavior as $\lambda_{\text{Exp}}(t)$ does not witness any deceleration. As a result the exponential DDC model does not capture the human mortality-rate pattern. On the other hand, the hazard rate of the DDC model with

gamma-distributed delays and no cure $\lambda_\Gamma(t)$ levels off, which corresponds to the observed mortality plateau for supercentenarians (Gampe 2010). It can be shown that distributions, which generate mortality plateaus when used as frailty distributions in the classical frailty model framework by Vaupel et al. (1979), contribute to patterns with the same behavior when taken as distributions of delay in a DDC model without cure. When cure is included into the model, results do not change significantly. A simulation study to analyze the effect of different delay distributions on the period age-specific mortality rates and life expectancy is yet to be performed.

References

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